# Mechanics and Relativity

**Dr. Umayal Ramanathan College for Women**
**Accredited with B+ Grade by NAAC**
**Affiliated to Alagappa University**
**(Run by Dr. Alagappa Educational Trust)**
**Karaikudi – 630 003**

**Study Material**
**Academic Year 2020-21**

**Name of the Faculty: Ms. MS. Kalaivani**
**Department: Physics**

**Course Outcomes**

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|---|---|---|
| **Mechanics and Relativity** | **7BPH1C2** | **CO1: Understand the definition for centre of gravity in solid hemisphere, hollow hemisphere .** |
| | | **CO2:Understand Projectile motion and the laws of gravitation.** |
| | | **CO3: Learn the basic terms and concepts of rigid body dynamics.** |
| | | **CO4: Understand the concepts like Centre of pressure, Floating bodies and Liquid motion.** |
| | | **CO5 : Realize the fundamentals of Special theory of Relativity and the related experiments.** |

**I YEAR – I SEMESTER**
**COURSE CODE: 7BPH1C2**

**CORE COURSE-II – MECHANICS AND RELATIVITY**

**Unit I STATICS**

Definition Centre of Gravity – solid hemisphere, hollow hemisphere – solid cylinder – tetrahedron – right solid cone. Friction – Laws of friction – Coefficient of friction – angle of friction – cone of friction – limiting friction – Equilibrium of a body on a rough inclined plane (free and forced) – Friction clutch.

**Unit II DYNAMICS AND GRAVITATION**

Projectiles **–** Path, Range and time of flight of a projectile and its applications.

Gravitation – Newton's law of gravitation – Kepler's laws of planetary motion – Newton's law from Kepler's law – Boy's method of finding G. Gravitational potential and intensity due to spherical shell and solid sphere – variation of 'g' due to height, depth and lattitude – escape velocity– motion of a rocket – orbital velocity – geostationary orbit.

**Unit III RIGID BODY DYNAMICS**

Definition of Moment of Inertia **–** Parallel and perpendicular axis theorems – Torque – Angular momentum– Conservation of linear and angular momentum – Kinetic energy of a rotating body. Compound pendulum – Centre of gravity and Centre of suspension – Theory of compound pendulum – Determination of g and k – Kater's pendulum.

**Unit IV HYDROSTATICS AND HYDRODYNAMICS**

Centre of Pressure**:** Definition – Centre of Pressure of rectangular and triangular laminae. Floating bodies**:** Law of floating bodies – Meta centric height – Meta centric height of a ship.

Equation of continuity – energy of liquid in motion – Bernoulli's theorem and its applications

**Unit V RELATIVITY**

Michelson Morley experiment and its importance – Postulates of special theory of relativity – Galilean and Lorentz transformations. Length contraction and time dilation – Addition of velocities – Einstein's mass energy equivalence.

**Text Books:**


1. Mechanics Part I and II – Naryanamoorthy, National Publishing Company, New Delhi, 2005
2. Mechanics – D.S.Mathur, S. Chand & Co, I Edition, New Delhi, 2006
3. Mechanics and Mathematical Methods – R.Murugeshan, S.Chand & Co, II Edition, New Delhi, 2005.
4. Modern Physics – R.Murugeshan, S. Chand & Co. (for Relativity),13th Edition, New Delhi, 2008.

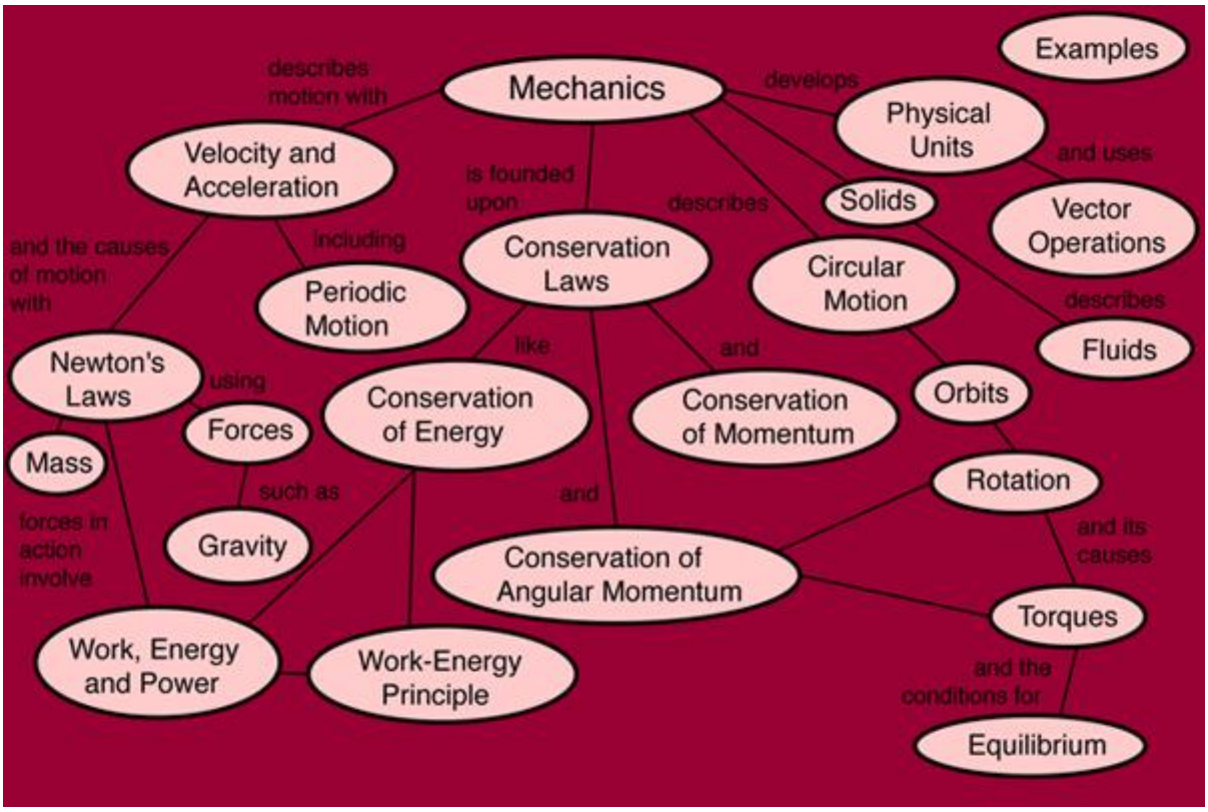

**Books for Reference:**


1. Fundamentals of Physics – D. Halliday, R. Resnick and J. Walker –, 6th Edition, Wiley, New York 2001
2. Mechanics and General Properties of Matter– P.K.Chakrabarthy, Books and Allied(P)Ltd. New Delhi,

**Course Objectives:**

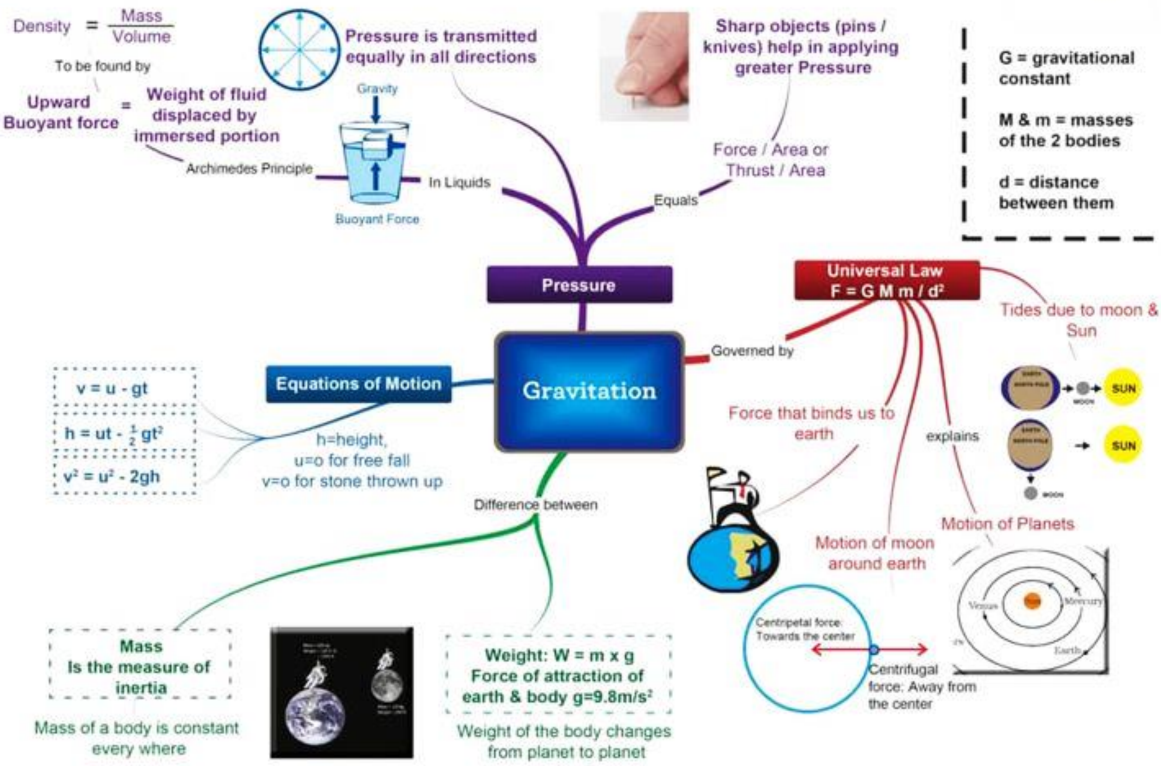
- Mechanics is a branch of the physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be subdivided into three branches: rigid-body mechanics and fluid mechanics.
- In this semester we will study rigid-body mechanics since it is a basic requirement for the study of the mechanics of deformable bodies and the mechanics of fluids. Rigid-body mechanics is divided into two areas: statics and dynamics.
- Statics deals with the equilibrium of bodies, that is, those that are either at rest or move with a constant velocity; whereas dynamics is concerned with the accelerated motion of bodies. We can consider statics as a special case of dynamics, in which the acceleration is zero; however, statics deserves separate treatment in education since many objects are designed with the intention that they remain in equilibrium
- Relativity has profoundly changed the whole physics. By the analysis of the fundamental concepts of space and time, of mass and of force, it has given a new orientation not only to science but also to our approach to philosophical problems in general.
- It is the theory which says that concepts like space, time, mass, simultaneity, motion etc., are not absolute but relative; absolute to frame of reference

**Concept Map of this Course**



# UNIT - I

## Mind Map



**Objectives**

Recognize the concept of friction and the expression for center of gravity in hemisphere, hollow hemisphere etc.

## 1. CENTRE OF GRAVITY

**Definition:** *The centre of gravity of a body is the point at which the resultant of the weights of all the particles of the body acts, whatever may be the orientation of the body. The total weight of the body may be supposed to act at its centre of gravity.*

Suppose the particles A,B,C........ of a body have masses m1, m2, m3......... Let their coordinates in a rectangular cartesian coordinate system be $(x_1,y_1,z_1)$, $(x_2,y_2,z_2)$,………..( $x_n,y_n,z_n$).

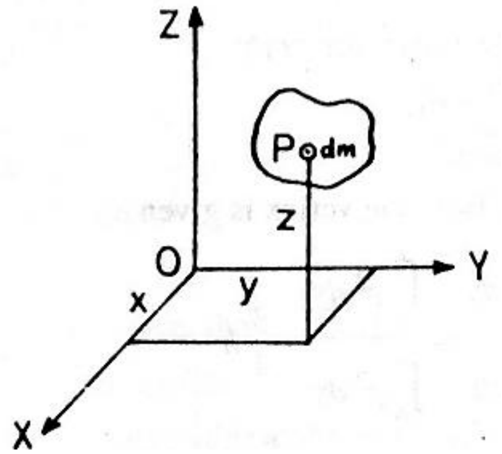



**Fig 1.1**

Then, the coordinates of the centre of gravity G of the body are

$$\bar{x} = \frac{\sum m_n x_n}{\sum m_n};$$

$$\bar{y} = \frac{\sum m_n y_n}{\sum m_n}; \qquad \bar{z} = \frac{m_n z_n}{\sum m_n};$$

Suppose an element P of the body has a mass (Fig) and its coordinates are x,y,z. Then,

$$\bar{x} = \frac{\int x\,dm}{\int dm} = \frac{1}{M}\int x\,dm\,;\ \bar{y} = \frac{1}{M}\int y\,dm\ ;\ \bar{z} = \frac{1}{M}\int z\,dm$$

Here, the integrals extend over all elements of the body, and $\boldsymbol{M = \int dm}$ = Total mass of the body.

### 1.1 Distinction between C. G and C. M.

1. Now weights of the different particles constituting the body are proportional to the respective masses. Hence, C.G.., if it exists is the same as the C.M..

2. If the body be removed to an infinite distance in space where the attracting force of the earth is inoperative or if it be imagined to be taken to the centre of the earth, the force of gravity there will be zero. The body will lose its weight. Hence, there arises no question of centre of gravity. But the body will have centre of mass as it will retain its mass which is in**dependent** of gravity and is an inherent property of matter. Thus a body *may not have a centre of gravity but it has a centre of mass*.

**1.2 Centre of gravity of a right solid cone:**

Let A*BC* represent a solid cone of height *h* and semi-vertical angle (In Fig). The cone may be considered to be made up of a large number of circular discs parallel to the base. The centre of gravity of each disc lies at its centre. Therefore, the C.G., of the cone should lie along the axis A*D o*f the cone.

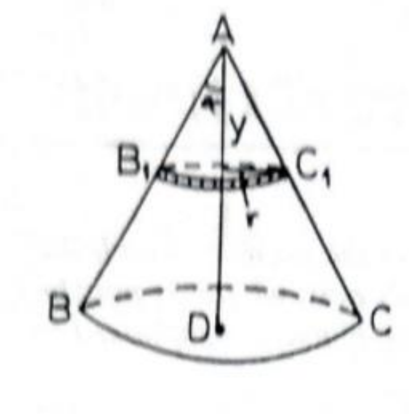



**Fig 1.2**

Consider a disc $B_1$ $C_1$ of thickness dy at distance y below the vertex A. If r is the radius of the disc, then

$$\boldsymbol{r = y\tan \propto}$$

Volume of the disc = Area × thickness = $\pi y^2 tan^2 \propto dy$

Mass of the disc = dm = $\pi y^2 \rho \ tan^2 \propto dy$

Where $\rho$ = density of the cone.

The distance of the C.G of the cone from the vertex is given by

$$\bar{y} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^h \pi y^3 \rho \tan^2\alpha\, dy}{\int_0^h \pi y^2 \rho \tan^2\alpha\, dy} = \frac{\int_0^h y^3\, dy}{\int_0^h y^2\, dy} = \frac{3}{4}h \cdot$$

Therefore, the C. G., of the cone is along its axis at a distance of *h* from the vertex.

### 1.3 Centre of gravity of a hollow right circular cone (without base)

Let h be the height of the cone. The slant surface of the cone may be divided into an infinite number of triangles A*BC*, A*CD*, ....*e*tc., by joining the vertex A to the points on the edge of the base (Fig).

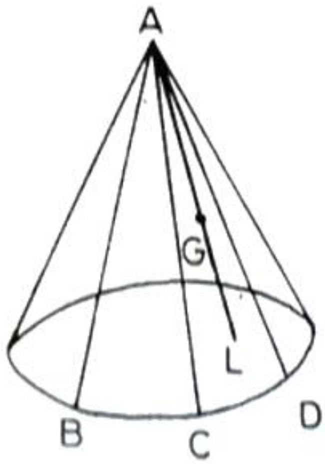



**Fig 1.3**

The centre of gravity of each such triangular area is at its centroid. It is at a height of *h*/3 above the circular base of the cone. Hence, the *C*. G., of the whole cone must lie on a plane parallel to the base at a height W/3 from it. By symmetry, the C. *G*., must also lie on the axis of the cone A*L*. Hence, the C. G., of the hollow cone is at G such that $\frac{GL}{AL} = \frac{1}{3}$

### 1.4 Centre of gravity of a solid hemisphere:

Let A*BC* represent a solid hemisphere of radius r, de centre O and density p (Fig). Consider an elementary slice of the hemisphere with radius y and thickness *d*x, at a distance x from *0*.

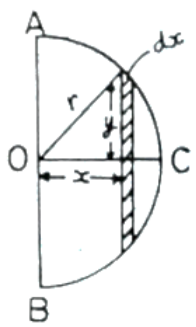



**Fig 1.4**

Volume of the slice = $\pi y^2\, dx\; = \pi\, (r^2 - \; x^2)\, dx.$

Mass of the slice = dm = $\rho\pi\; (r^2 - \; x^2)\, dx.$

The distance of the C.G., of the hemisphere from O is given by

$$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^r x\, \rho\pi\, (r^2 - x^2)\, dx}{\int_0^r \rho\pi\, (r^2 - x^2)\, dx} = \frac{\int_0^r (r^2 x - x^3)\, dx}{\int_0^r (r^2 - x^2)\, dx}$$

$$\bar{x} = \frac{3}{8} r.$$

Hence, the C.G., of the solid hemisphere is on its axis at a distance 3r/8 from the centre.

**1.5 Centre of gravity of a hollow hemisphere.**

Let A*CB* be a section of a hemisphere of radius r,centre *O* and surface density p [Fig. 3.5]. Imagine the surface of the hemisphere to be divided into slices like *PQQ,P*, by planes parallel to A*B*. If ∟POC = θ and ∟POC = dθ, then

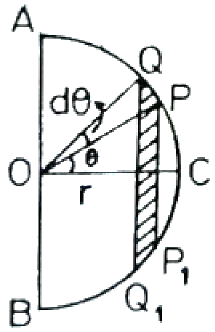



**Fig 1.5**

Radius of the ring = r sinθ

Width of the ring = r dθ

Area of the ring = 2π r sinθ. r dθ

Mass of the ring = dm = 2π $r^2$ ρ sinθ. dθ

The C.G., of this ring is at the centre of the ring at a distance rcosθ from O.

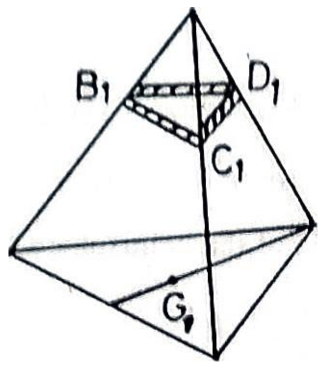
The distance of the C.G., of the hollow hemisphere from O is given by

Fig

$$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^{\pi/2} (r \cos\theta)\, 2\pi r^2\, \rho \sin\theta\, d\theta}{\int_0^{\pi/2} 2\pi r^2\, \rho \sin\theta\, d\theta} = \frac{\int_0^{\pi/2} r \sin\theta \cos\theta\, d\theta}{\int_0^{\pi/2} \sin\theta\, d\theta}$$

$$\therefore \qquad \bar{x} = r/2\,.$$

The C.G., of a hollow hemisphere is on its axis at a distance r/2 from the centre, i.e., the centre of gravity is at the midpoint of the radius OC.




### 1.6 Centre of gravity of solid tetrahedron

Let ABCD be the tetrahedron and $G_1$ the centre of gravity of the base BCD (Fig.), Let h be the altitude of the tetrahedron and ρ its density. Suppose the tetrahedron is divided into thin slices by planes parallel to the base BCD. Consider one such slice $B_1$ $C_1$ $D_1$ of thickness dx at a depth x below A, Let S be the area of the triangular base BCD.

A

Then we have, $\frac{B_1C_1}{BC} = \frac{x}{h}$.

If $a_1$ and a are the altitudes of triangles $B_1C_1D_1$ and BCD respectively.

C

B $\frac{a_1}{a} = \frac{x}{h}$.

Now, area of $\Delta\ B_1C_1D_1 = \frac{1}{2}\ B_1C_1 \times a_1$

area of $\Delta$ BCD $= \frac{1}{2}$ BC × a

Hence, $\frac{Area\ of\ \Delta\ B_1C_1D_1}{s} = \frac{B_1C_1}{BC} \times \frac{a_1}{a} = \frac{x^2}{h^2}$

Therefore, Area of $\Delta\ B_1C_1D_1 = Sx^2/h^2$

Volume of the slice $B_1C_1D_1 = Sx^2\,dx/h^2$

Mass of the slice = dm = $\rho\ Sx^2\,dx/h^2$.

The distance of the centre of gravity of the tetrahedron from A is given by

$$\bar{x} = \frac{\int x\, dm}{\int dm} = \frac{\int_0^h x \rho\, S x^2\, dx/h^2}{\int_0^h \rho\, S\, x^2\, dx/h^2} = \frac{\int_0^h x^3\, dx}{\int_0^h x^2\, dx} = \frac{3}{4} h$$

Hence, the C.G., of a uniform tetrahedron lies at a point G on the line AH such that AG : GH = 3 :1.

## 1.7 Centre of gravity of a solid cylinder

Let m be the mass of the solid cylinder

Let R be the radius of the solid cylinder

h = height of the solid cylinder

Consider a cylinder is composed of large number of solid disc. Let us consider small elementarystrip of thickness dy and it is placed at a distance of y from the top of the cylinder.

Let dv be the volume of the elementary strip.

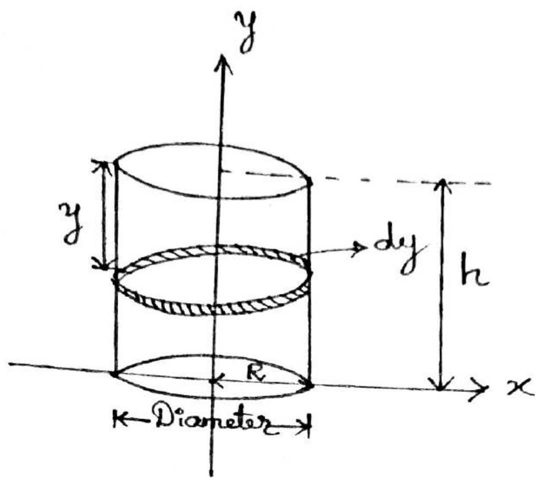



**Fig 1.7**

$$\bar{y} = \frac{\int y\,.\,dv}{\int dv}$$

$dv = \pi R^2\, dy$

dy = thickness of the elementary strip.

$$\bar{y} = \frac{\int_0^h y\,\pi R^2\, dy}{\int_0^h \pi R^2\, dy} = \frac{\int_0^h y\, dy}{\int_0^h dy}$$

$$= \frac{\frac{h^2}{2}}{h} = \frac{h^2}{2} \times \frac{1}{h} = \frac{h}{2}$$

Hence for a cylinder, centre of gravity lies at the midpoint of the axis of the cylinder.

## 2.1 Friction

### 2.1.1 Introduction:

Suppose we push a book along a table, giving it some velocity. After we release it, it slows down and finally stops. That indicates that a force is opposing the motion. Whenever one surface moves in contact with another, a force is generated that tends to retard the motion, This force is called **friction**.

riction plays a vital role in our life. For example, man is able to walk on the road because of friction between his feet and the ground. In machines friction reduces efficiency. Lubrication and ball bearings reduce friction In machines.

### 2.1.2 Static, Dynamic, Rolling and Limiting Friction:

The frictional forces acting between surfaces at rest with respect to each other are called forces of static friction. Tie maximum value of the frictional force between two bodies in contact is called limiting friction. The forces acting between surfaces in relative motion are called kinetic (or dynamic) friction. The frictional force between two surfaces when one rolls over the other is called rolling friction. It is easy to roll a cylindrical body than to slide it. Thus, the rolling friction is less than the sliding friction We require more force to start the motion of a body than to keep it in uniform motion. Thus, static friction is larger than dynamic friction.

### 2.1.3 Laws of Static Friction:

1. The direction of the frictional force is always opposite to the direction in which one body tends to slide over another.
2. The magnitude of the force of friction when there is equilibrium between two bodies is just sufficient to prevent the motion of one body over the other. The frictional force attains a maximum value when one body is just on the point of sliding over the other. The maximum value of the force of friction is called limiting friction.
3. The magnitude of the force of limiting friction bears a constant ratio to the normal reaction between the two bodies. This ratio is called c*oefficient of friction* and is denoted by $\mu$. If *F* is the limiting friction and R the normal reaction between the two bodies, then $\mu = F/R$. $\mu$ depends only on the nature of surfaces in contact.

4. The limiting friction is independent of the extent and shape of the surfaces in contact provided the normal reaction is unaltered.
5. When a body is in motion, the direction of friction is still opposite to the direction of motion of the body and is independent of the velocity. But the ratio of the force of friction to the normal reaction is slightly less than that when the body is just on the point of motion.

**2.1.4 Angle of friction:**

Let F be the force of limiting friction and R, the normal reaction. Let S be the resultant of these two forces. Then the angle which this resultant force makes with the normal reaction is called the angle of friction. It is denoted by λ.

$$\text{Then, } \tan\lambda = \frac{F}{R} = \frac{\mu R}{R} = \mu$$

**2.1.5 Cone of friction:**

Consider a cone with the point of contact of two bodies are the vertex, the normal reaction as axis and semi-vertical angle λ. Then the resultant reaction (S) may lie anywhere within or on the surface of the cone. This imaginary cone is called the *cone of friction.*

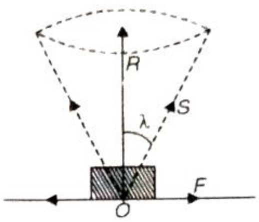



**Fig 2.1**

**2.1.6 Experimental Method for Determining Co-efficient of Friction between two Surfaces**

Suppose a body of weight *mg* is placed on a rough inclined plane. The inclination is increased till a position is attained when the body just slides. At this position let the inclination of the plane to the horizontal be α. The forces acting on the body are: (i) the weight mg acting vertically downwards. *(ii)* the normal reaction R perpendicular to the plane and (iii) the force of limiting friction μR acting up the plane. Resolve *mg* into a component *mg cosα* at right angles to the plane. When the equilibrium is limiting.

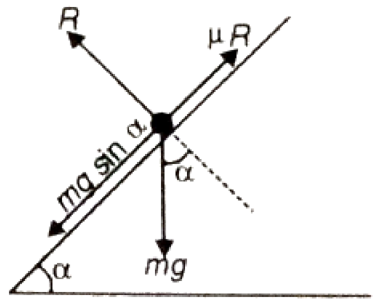



**Fig 2.2**

$$mg \sin \alpha = \mu R$$

$$mg \cos \alpha = R$$

Dividing (i) and (ii), $\tan \alpha = \mu$

### 2.1.7 Equilibrium of a body on a rough inclined plane acted upon by an external force

**PROPOSITION:** A body of weight w is in equilibrium on a rough inclined plane of angle $\alpha > \lambda$ under the action of an external force inclined upwards at an angle $\theta$ with the plane. Find the value of P for limiting equilibrium.

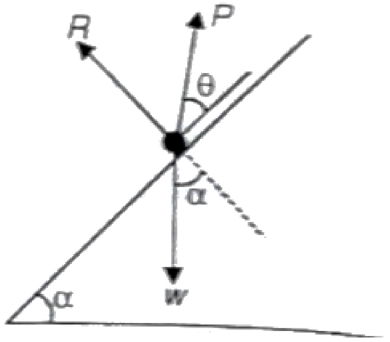



**Fig 2.3**

**Case 1:** Let the body be just on the point of sliding down the plane. Let P be the magnitude of the of the external force, applied at an angle $\theta$ with the plane. The forces acting on the body are: (i) The weight of the body (w) acting vertically down, (ii) The normal reaction (R) acting perpendicular to the plane (iii) The force of limiting friction ($\mu R$) acting up the plane and (iv) The external force (effort) P making an angle $\theta$ with the line of greatest slope of the inclined plane. Resolving the forces parallel and perpendicular to the plane,

$$P \cos\theta + \mu R = w \sin\alpha \quad \text{------------------------------------} \ (1)$$

$$P \sin\theta + R = w \cos\alpha \quad \text{------------------------------------} \ (2)$$

Multiplying (2) by μ and subtracting from (1),

$$P (\cos\theta - \mu \sin\theta) = w (\sin\alpha - \mu \cos\alpha)$$

$$P = w \frac{(\sin\alpha - \mu\cos\alpha)}{(\cos\theta - \mu\sin\theta)}$$

But $\mu = \tan\lambda$, where λ is the angle of friction.

$$\text{Hence} \qquad P = w \frac{(\sin\alpha - \tan\lambda\ \cos\alpha)}{(\cos\theta - \tan\lambda \sin\theta)}$$

$$P = w \frac{(\sin\alpha \cos\lambda - \sin\lambda\ \cos\alpha)}{(\cos\theta\cos\lambda - \sin\lambda\sin\theta)}$$

$$P = w \frac{\sin(\alpha - \lambda)}{\cos(\theta + \lambda)} \quad \text{------------------------------------}(3)$$

**Case 2:** Let the body be just on the point of sliding up the plane. Let $P_1$ be the magnitude of the external force. In this case, the force of limiting friction (μR) acts down the plane. Resolving the forces parallel and perpendicular to the plane.

$$P_1 \cos\theta = w \sin\alpha + \mu R \quad \text{-------------------------------}(4)$$

$$P_1 \sin\theta + R = w \cos\alpha \quad \text{------------------------------------}(5)$$

$$\text{Simplifying, we get } P_1 = w \frac{\sin(\alpha + \lambda)}{\cos(\theta - \lambda)}$$

**Corollary 1:** $P_1$ is a minimum when $\cos(\theta - \lambda)$ is maximum i.e., when $\cos(\theta - \lambda) = 1$ i.e., $\theta = \lambda$. Hence force required to move the body up the plane will be least when it is applied in a direction making with the inclined plane an angle equal to the angle of friction.

**Corollary 2:** Let a body be at rest on a rough inclined plane whose inclination to the horizontal $\alpha > \lambda$. Let it be acted upon by an external force applied parallel to the plane. Here $\theta = 0$. From (3) and (6),

$$P = w \frac{\sin(\alpha - \lambda)}{\cos\lambda} \quad \text{-----------------------------------------}(7)$$

$$P_1 = w \frac{\sin(\alpha + \lambda)}{\cos\lambda} \quad \text{-----------------------------------------}(8)$$

**Summary:**

This unit explained that the centre of gravity in

- solid hemisphere – 3r/8
- hollow hemisphere – r/2
- solid cylinder – h/2
- tetrahedron – 3h/4
- right solid cone – 3h/4

This chapter provides that the Friction, Laws of friction, Coefficient of friction, angle of friction, cone of friction, limiting friction, Equilibrium of a body on a rough inclined plane, Friction clutch.

**Sample Question**

**2 Marks**

**1.** Define the co – efficient of friction.
**2.** What is limiting friction?
**3.** State any two laws of static friction.
**4.** Define cone of friction.
**5.** Define Centre of gravity.
**6.** What is meant by angle of friction?
**7.** What is static and dynamic friction?
**8.** Define angle of friction.
**9.** Distinguish between Centre of gravity and Centre of mass.

**5 Marks and 10 Marks:**

**1.** Obtain the expression for the Centre of gravity of a solid hemisphere.
**2.** Find the position of the centre of gravity of a uniform solid right circular cone.
**3.** Calculate the C.G of right solid cone.
**4.** Explain
**a.** Angle of friction
**b.** Co-efficient of friction
**c.** Cone of friction
**5.** Calculate the center of gravity of a hollow hemisphere.
**6.** Define the centre of gravity of a tetrahedron.

**7.** Discuss the equilibrium of a body on a rough inclined plane with the application of external force.

**8.** Find an expression for the force of friction of a body placed on a rough inclined plane.

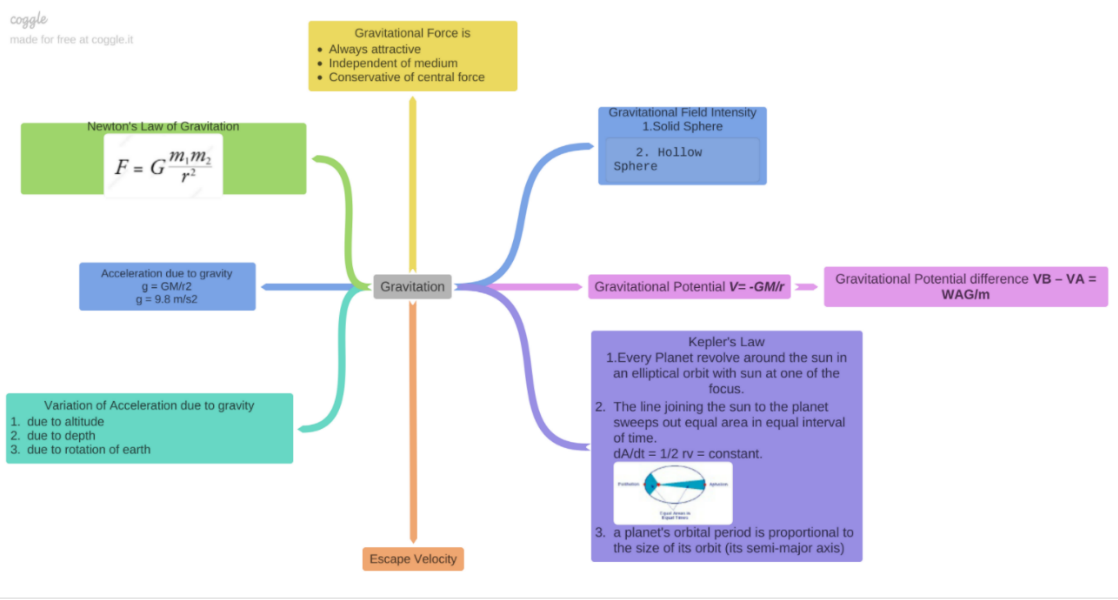
**9.** Write a note on friction clutch.

**10.** Discuss the equilibrium of a body on a rough inclined plane.

**11.** Define friction. Give the laws of friction.

# Unit - II

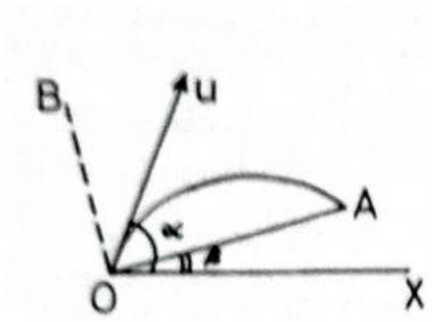
## Mind Map



**Learning Objectives**

Understand the projectile motion and the laws of gravitation

**2.1 Range on an inclined plane**




A particle projected with a velocity u at an angle to the horizontal from a point O on an inclined plane, inclined at an ß angle to the horizontal. The direction of projection lies in the vertical plane through OA the line of greatest slope, of the plane. Let the particle strike the inclined plane at A. Then OA (=R) is the range on the inclined plane.

Let OX and OA be respectively the horizontal and inclined plane through the point of projection O, OB is a line perpendicular to OA,

Component of initial velocity u along OA = u cos (a - ß) Component Of initial velocity u along OB = u sin (a - ß) The projectile moves with a vertical retardation g,

Acceleration along OA = -g gin **ß**

Acceleration along OB = -g cos **β**

Now, let T be the time taken by the particle to go from O to A. When the particle reaches A after time T, the distance moved perpendicular to the plane is zero.

Hence, $0 = u\sin(\alpha - \beta), T - \frac{1}{2} g\cos\beta, T^2 \left(\because s = ut + \frac{1}{2}at^2\right)$

$$\therefore T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

When the particle strikes $A$ after time $T$, the distance $OA(= R)$ moved is the range on the inclined plane.

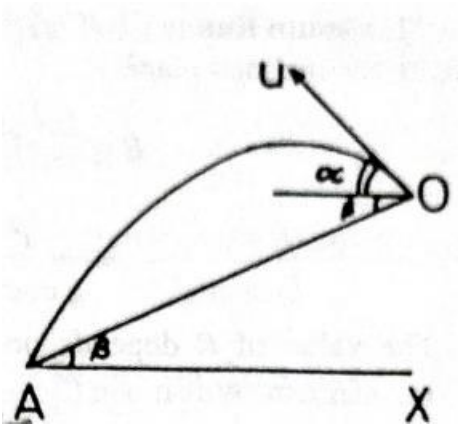
$$\therefore R = u\cos(\alpha - \beta), T - \frac{1}{2} g\sin\beta, T^2$$

$$= u\cos(\alpha - \beta)\frac{2u\sin(\alpha-\beta)}{g\cos\beta} - \frac{1}{2} g\sin\beta \frac{4u^2\sin^2(\alpha-\beta)}{g^2\cos^2\beta}$$

$$= \frac{2u^2 \sin(\alpha-\beta)}{g\cos^2\beta}[\cos(\alpha-\beta)\cos\beta - \sin(\alpha-\beta)\sin\beta]$$
$$= \frac{2u^2 \sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

**2.2. Range and Time of fight down an inclined plane**

The particle is projected down the plane from $O$ at an elevation $\alpha$. Initial velocities along and perpendicular Initial velocities along and perpendicular to $OA$ are $u\cos(\alpha+\beta)$ and $u\sin(\alpha+\beta)$. $OA$ are $g\sin\beta$ and $-g\cos\beta$. When the particle reaches $A$ after time $T_1$, the distance moved perpendicular to the in- 4 clined plane is zero. Therefore,




$$0 = u\sin(\alpha+\beta).T_1 - \frac{1}{2}g\cos\beta.T_1{}^2 \text{ or } T_1 = \frac{2u\sin(\alpha+\beta)}{g\cos\beta}$$

$$\text{Range} = = OA = \underbrace{R_1} = u\cos(\alpha+\beta).T_1 + \frac{1}{2}g\sin\beta.T_1{}^2$$

$$= \frac{2u^2\sin(\alpha+\beta)\cos\alpha}{g\cos^2\beta}$$

**Note 1:** Results down the palne can be obtained by putting -β for β in the results of the previous article.

**Note 2:** In some problems, the elevation relative to the inclined plane may be given. In such cases we must calculate the elevation relative to the horizontal.

**2.3 Newton's Law of Gravitation**

Statement : Every particle of matter in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

**Explanation :** If $m_1$ and $m_2$ are the masses of two particles situated at a distance $r$ apart, the force of attraction between them is given by

$$F \propto \frac{m_1 m_2}{r^2} \text{ or } F = \frac{G m_1 m_2}{r^2}$$

where $G$ is a universal constant, called the Universal gravitational constant. The law of gravitation is universal. It holds from huge interplanetary distances to extremely small distances. The law does not hold good for interatomic distances, which are as small as $10^{-9}$ m. The -force of attraction between any two bodies is not affected by the intervening medium. This force is also not affected by the nature, state or chemical structure of the bodies involved but depends only on their masses. Even temperature has no appreciable effect on gravitation.

**Definition of G.** If $m_1 = m_2 = 1$ kg and $r = 1$ m, then $F = G$. Thus, the Gravitational constant is equal to the force of attraction between two unit masses of matter unit distance apart.

**Dimensions of G.** $G = \frac{Fr^2}{m_1 m_2}$.

**Mass and Density of earth :** If $m$ is the mass of a body and $g$ the acceleration due to gravity, the force of attraction of the earth on the mass $m = mg$.

Let $M =$ mass of the earth ; $R =$ radius of the earth .

$$\left.\begin{array}{l}\text{Gravitational force of attraction between}\\ \text{a body of mass } m \text{ and earth}\end{array}\right\} = \frac{GMm}{R^2}$$

$$\frac{GMm}{R^2} = m, g \text{ or } M = \frac{R^2, g}{G}$$

Volume of the earth $= V = \frac{1}{3}\pi R^3$.

Density of the earth $= \rho = \frac{M}{V} = \frac{(R^2 g/G)}{\frac{4}{3}\pi R^3} = \frac{3\dot{g}}{4\pi RG}$

**Inertial Mass:** The mass of a body may be determined by measuring the acceleration a produced on it by a known force F. Thus, $m = F/a$. The mass $m$ thus determined is called inertial mass.

**Gravitational Mass:** The mass of a body may also be determined by measuring the gravitational force exerted on it by earth.

$$F = \frac{GMm}{R^2} \text{ or } m = \frac{FR^2}{GM}.$$

The mass m thus determined is called gravitational mass.

**2.4 Kepler's Laws of planetary motion**

(1) Every planet moves in an elliptical orbit around the sun, the sun being at one of the foci.

(2) The radius vector, drawn from the sun to a planet sweeps out equal areas in equal times i.e., the areal velocity of the radius vector is constant ($dA/dt =$ constant ).

(3) The square of the period of revolution of the planet around the sun is proportional to the cube of the semi-major axis of the ellipse ($T^2 \propto a^3$).

Deduction of Newton's Law of Gravitation from Kepler's Laws

Consider two planets of masses $m_1$ and $m_2$. Let $r_1$ and $r_2$ be the radii of their circular orbits. Let $T_1$ and $T_2$ be their periods of revolution round the sun.

The centrifugal force acting on the first planet,

$$F_1 = m_1 r_1 \cdot \omega^2 = m_1 r_1 \left(\frac{2\pi}{T_1}\right)^2$$

Similarly, the centrifugal force acting on the second planet

$$F_2 = m_2 r_2 \left(\frac{2\pi}{T_2}\right)^2$$

$$\frac{F_1}{F_2} = \frac{m_1 r_1}{m_2 r_2} \left(\frac{T_2}{T_1}\right)^2$$

But according to Kepler's third law, $\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$

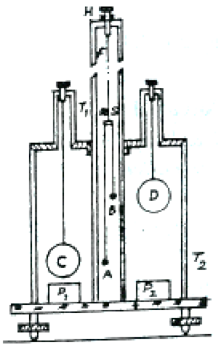
$$\frac{F_1}{F_2} = \frac{m_1 \cdot r_1}{m_2 \cdot r_2} \left(\frac{r_2}{r_1}\right)^3 = \frac{m_1 \cdot r_2^2}{m_2 \cdot r_1^2}$$

i.e., the force on the planet is directly proportional to $\frac{m}{r^2}$ or $F \propto \frac{m}{r^2}$. Therefore, the force is proportional to the mass of the planet. Since the attraction is mutual, the force is also proportional to the mass of the sun $M$. Hence $F \propto \frac{Mm}{r^2}$ or $F = \frac{GMm}{r^2}$ which is Newton's Law of Gravitation.

**2.5 Determination of G-Boys' Experiment**

The apparatus consists of two co-axial glass tubes $T_1$ and $T_2$ mounted on a platform provided with levelling screws (Fig. 6.1). The inner tube $T_1$ is fixed, while the outer tube $T_2$ can be rotated about the common axis. A small mirror, $RS$, is suspended in the inner tube by a fine quartz fibre $F$ from a torsion head $H$.

From the two ends of the mirror, two gold spheres $A$ and $B$ are suspended, such that the spheres are at different depths below the mirror. In the outer co-axial tube $T_2$, two large lead balls $C$ and $D$ are suspended from its revolving lid such that the centre of $C$ is in level with that of $A$, the centre of $D$ is in level with that of $B$ and the distance $AC = BD$. Two rubber pads $P_1$ and $P_2$ are placed below the two lead spheres, as a safeguard against damage, in case they should fall accidentally.



The experiment is performed by rotating the outer glass tube until the large lead spheres lie on the opposite sides of the two gold balls, so as to exert the maximum moment on the suspended system. In this position, the angle through which the mirror $(RS)$ turns is maximum. The outer glass tube is then rotated so that the lead spheres now lie on the other sides of the gold balls, in an exactly similar position, producing the greatest deflection. The mean of these two observations gives the deflection of the mirror $\theta$.

A lamp and scale arrangement is used to measure $\theta$.

Force of attraction between spheres. $A$ and $C = GMm/(AC)^2$

Force of attraction between spheres $B$ and $D = GMm/(BD)^2$

Since $AC = BD$, the two forces are equal, parallel and act in opposite directions, thus constituting a couple.

$$\therefore \left.\begin{matrix}\text{The moment of the} \\ \text{deflecting couple}\end{matrix}\right\} = \frac{GMm}{(AC)^2} \times 2l = \frac{GMm}{d^2} \times 2l$$

(where $2l =$ the length of the mirror strip $RS$ and $AC = d$ ).

The deflection of the mirror strip under this couple is resisted by the torsion or twist set up in the suspension fibre. The mirror strip comes to rest when the deflecting couple due to gravitational pull is balanced by the restoring torsional couple set up in the suspension fibre. Now, if $c$ be the torsional couple per unit twist, then for angular deflection $\theta$, the total restoring couple is $c.\theta$.

$\therefore$ In equilibrium position, $\frac{GMm}{d^2} \times 2l = c\theta$.

From this, the value of $G$ can be calculated. Using the arrangement of the quartz fibre and the mirror strip with gold balls as a torsion pendulum, the period $T$ is found. Then $T = 2\pi\sqrt{I/c}$ where $I =$ moment of inertia of the suspended system. From this $c$ can be calculated.

The results obtained by him are very accurate. The value obtained for $G$ by Boys is $6.6576 \times 10^{-11}\mathrm{Nm^2\ kg^{-2}}$

Advantages :

(1) The size of the apparatus is very much reduced. The disturbances due to convection currents are therefore almost negligible.

(2) By arranging the masses at different levels, the effect of the attraction of the heavier mass on the remote smaller mass is very much reduced.

(3) By the lamp and scale arrangement, very small deflections can be measured accurately.

(4) The use of a quartz fibre has made the apparatus very sensitive and accurate.

### 2.6 Gravitational Field and Gravitational Potential

**Gravitational Field:** *The space around a body within which its gravitational force of attraction is perceptible is called its gravitational field.*

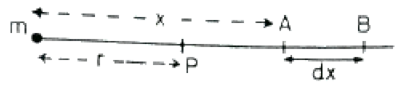
The gravitational field is an example of a vector field. Each point in this field has a vector associated with it. The intensity of the gravitational field at a point due to a body is the force experienced by a unit mass placed at that point.

**Gravitational Potential**: *The work done in moving a unit mass from infinity to a point in a gravitational field is called the gravitational potential at that point.*

Gravitational potential is always negative in sign, its highest value being zero at infinity. It is a scalar quantity.

**Intensity of gravitational field at a point** : It is defined as the space rate of change of gravitational potential at the point. i.e., $F = -\frac{dV}{dr}$ where $dV$ is the small change of gravitational potential for a small distance $dr$.

**Gravitational potential due to a point mass**. Consider a point $A$ at a distance $x$ from a particle of mass $m$.




Force of attraction experienced by a unit mass at A = $\frac{G.m}{x^2}$

Work done in displacing the unit mass from A to B through a distance dx = $\frac{G.m}{x^2}\ dx$

∴ The potential difference between $A$ and $B = \delta V = \frac{G\cdot m}{x^2} dx\ P$ or

The potential at $P = V = \int \delta V = \int_{\infty}^{r} \frac{G\cdot m}{x^2} dx = -\frac{G\cdot m}{r}$

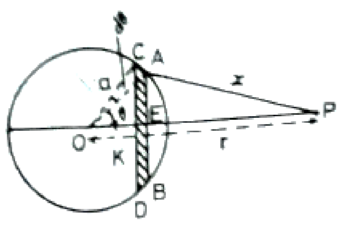
Thus the gravitational potential has the maximum value of zero at infinity and decreases as the distance is decreased.

**Equipotential Surface**: *A surface at all the points of which the gravitational potential is the same is called an equipotential surface*. For example, a spherical surface around a point mass with the mass as centre, is an equipotential surface. Since the potential on this surface is constant, no work is done against the gravitational force in moving a unit (or any other) mass alongit.

**2.7 Gravitational Potential and Field due to a Spherical Shell**

**(i) Point outside the shell:** Consider a point $P$ outside the spherical shell at a distance $r$ from its centre $O$ (Fig. 6.3). Let $a$ be the radius of the shell, $\rho$ the mass per unit area of the surface of the shell, and $M$ its total mass. Join $OP$ and let $OP = r$. Consider a thin slice of the shell contained between two planes $AB$ and $CD$ drawn close to each other at right angles to $OP$. Join $O$ and $A, O$ and $C$ and $A$ and $P$.

Let $\angle AOP = \theta$, and $\angle AOC = d\theta$.




Now, $AE =$ Radius of the slice $= a\sin\ \theta$

Circumference of the slice $= 2\pi \times AE = 2\pi a\sin\ \theta$.

Width of the slice $= CA = ad\theta$.

Hence, surface area of the slice $= 2\pi a\sin\ \theta \times ad\theta$

$= 2\pi a^2 \sin\ \theta d\theta$.

∴ Mass of the slice $= 2\pi a^2 \rho \sin\ \theta d\theta$.

et $PA = x$. Every point on the slice may be taken to be practically equidistant from $P$,

$\therefore$ $\left.\begin{array}{l}\text{Potential at } P\\ \text{due to the ring}\end{array}\right\} = dV = \dfrac{-G2\pi a^2 \rho \sin\ \theta d\theta}{c/x}$

To find the value of $x$, consider the triangle $OAP$. $x^2 = a^2 + r^2 - 2ar\cos\ \theta$

Differentiating, $2xdx = 2ar\sin\ \theta d\theta$ [$\because$ $a$ and $r$ are constants ] or $x = \frac{a \cdot r\sin\ \theta d\theta}{dx}$

$$dV = \frac{-G2\pi a^2 \rho \sin\ \theta d\theta dx}{a \cdot r\sin\ \theta d\theta} = \frac{-2\pi a \cdot \rho G}{r} dx.$$

If the entire shell is split up into slices of this kind, the value of $PA$ will vary from $(r-a)$ to $(r+a)$.

Hence,

$\left.\begin{array}{l}\text{the potential at } P \text{ due}\\ \text{to the entire shell}\end{array}\right\} = V = \int_{r-a}^{+a} \frac{-2\pi a \rho G}{r} dx.$

$$. = \frac{-2\pi a \cdot \rho G}{r} [x]_{r-a}^{r+a} = \frac{-2\pi a \rho G}{r} \cdot 2a = -4\pi a^2 \rho \frac{G}{r}$$

Now $4\pi a^2 \rho =$ Mass of the whole shell.

$$V = -\frac{G, M}{r}$$

This potential is the same as due to a mass $M$ at $O$. Hence, the mass of the shell behaves as though it were concentrated at its centre.

(ii) Point on the surface of the shell. Let us consider a point which lies on the surface of the shell itself. The limits for the value of $x$ will be 0 and $2a$. Hence

$\left.\begin{array}{l}\text{Potential at a point on}\\ \text{the surface of the shell}\end{array}\right\} = V = \int_0^{2a} \frac{-2\pi a \rho G}{r} dx = \frac{-2\pi \cdot a \cdot \rho \cdot G}{r} [x]_0^{2a} = \frac{-4 \cdot \pi \cdot a^2 \rho \cdot G}{r} = \frac{-G \cdot M}{r} = \frac{-G \cdot M}{a}$; $(\because r = a \therefore\ V = \frac{-G \cdot M}{a}$

(iii) Point inside the shell. Let the point $P$ be situated at $K$ inside the shell, such that $OK = r$. The limits for the value of $x$ will be $(a-r)$ and $(a+r)$.

$\therefore$ $\left.\begin{array}{l}\text{Potential at a point } (K)\\ \text{inside the shell}\end{array}\right\} = V = \int_{a-r}^{a+r} \dfrac{-2\pi a\rho \cdot G}{r} dx$

$$= \frac{-2\pi a \rho \cdot G}{r} [x]_{a-r}^{a+r} = -4\pi a \rho \cdot G$$

Multiplying and dividing by $a, V = \frac{-4\pi a^2 \rho \cdot G}{a} = \frac{-G \cdot 1}{a}$

$\therefore V = -\frac{G \cdot M}{a}$

Hence the potential at all points inside a spherical shell is the same and is equal to the value of the gravitational potential on the surface.

GRAVITATIONAL FIELD. The intensity of the gravitational field $F$ is given by $F=-dV/dr$.

(i) At a point outside the shell : $V=\frac{-GM}{r}$

$$\therefore\ F=\frac{-dV}{dr}=-\frac{d}{dr}\left[\frac{-G\cdot M}{r}\right]=\frac{-G\cdot M}{r^2}$$

The negative sign indicates that the force is towards the centre $O$

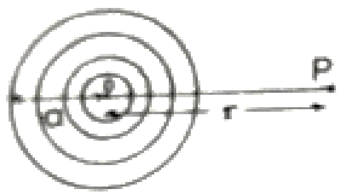
(ii) At a point on the outer surface of the shell : Putting $r=a$ in the expression (i), we get the intensity of the gravitational field at a vint on the surface of the shell.

$F=-\frac{GM}{a^2}$ inside the shell. ial $V=\frac{-GM}{a}=a$ constant $\therefore F=\frac{-dV}{dr}=0$

(iii) At a point inside the shell. Potential $V=\frac{-GM}{a}=$ a constant. $\therefore F=\frac{-dV}{dr}$ =0

Gravitational Potential and Field due to a solid sphere

(i) Point outside the sphere. Let $P$ be a point outside the sphere at a distance $r$ from the centre $O$ [Fig. $(a)$.




Let $M$ be the mass $P$ of the sphere, $a$ its radius and $\rho$ its density. A solid sphere may be imagined to be made up of a large number of concentric shells. Each one of the shells produces a potential at thsoint $P$ outside the shell, as if its entire mass Fig. $6.4(a)$ is concentrated at the centre $O$. Thus if $m$ is the mass of one such shell,

The potential at $P$ due to the shell $=\frac{-Gm}{r}$

Potential due to the whole sphere $V=-\Sigma\frac{Gm}{r}=-\frac{G}{r}\Sigma m$

Clearly, $\Sigma m=M=$ Mass of the solid sphere.

\frac{-G M}{r}

(ii) Point on the surface : If the point $P$ lies on the surface of the solid sphere, we have $r=a$. Putting $r=a$ in $(i)$, we get, The potential at a point on the surface $= \frac{-GM}{a}$

(iii) Point inside the sphere : Let the point now lie inside the solid sphere at a distance $r$ from the centre $O$.

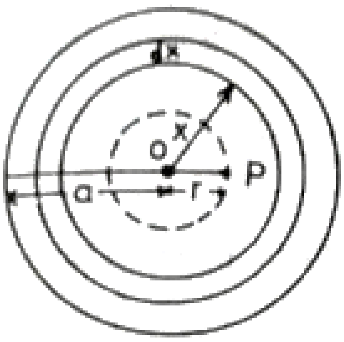
The solid sphere may be imagined to be made up of $(t)$ an inner solid sphere of radius $r$ and (ii) a hollow sphere of internal radius $r$ and external radius $a$. The hollow sphere may be imagined to be made up of concentric shells with radii ranging from $r$ to $a$.

Potential at $P$ due to the whole solid sphere-is

$V =$ potential at $P$ due to the inner solid sphere $(V_1)$ + potential at $P$ due to all the shells $(V_2)$

(a) To determine the potential at P due to the inner solid sphere

The point $P$ lies on the surface of the inner solid sphere of radius $r$ [Fig. 6.4 $(b)$].




Mass of the inner sphere $= \frac{4}{3}\pi r^3 \rho$.

Potential at $P$ due to the inner sphere

Fig. 6.4$(b)$

$$= V_1 = \frac{-G\frac{4}{3}\pi r^3 \rho}{r} = -G\frac{4}{3}\pi r^2 \rho \ldots (i)$$

(b) To determine the potential at P due to all the outer shells Consider one such shell of radius $x$ and thickness $dx$. The point $P$ lies inside the spherical shell.

Mass of the shell $= 4\pi x^2 dx\rho$.

$\therefore$ Potential at $P$ due to this shell $= \dfrac{-G4\pi x^2 dxp}{x} = -G4\pi x dxp$

$\therefore$ $\left.\begin{array}{l}\text{Potential at } P \text{ due}\\ \text{to all shells}\end{array}\right\} = V_2 = \int_r^a -G4\pi x dxp = -G4\pi\rho\int_r^a x dx = -G4\pi\rho\left[\frac{x^2}{2}\right]_r^a = -G4\pi\rho\left(\frac{a^2-r^2}{2}\right)$

$\therefore$ Total potential at $P = V = V_1 + V_2 = -G\frac{4}{3}\pi r^2\rho - G4\pi\rho\left(\frac{a^2-r^2}{2}\right) = -G\frac{4}{3}\pi\rho\left[r^2 + \frac{(3a^2-3r^2)}{2}\right] =$ $-G\frac{4}{3}\pi\rho\left[\frac{2r^2+3a^2-3r^2}{2}\right] = -G\frac{4}{3}\pi\rho\left[\frac{3a^2-r^2}{2}\right] = -G\frac{4}{3}\pi a^3\rho\left[\frac{3a^2-r^2}{2a^3}\right]$ (multiplying and dividing by $a^3$ ) $\therefore$

$$= -G\frac{4}{3}\pi r^2\rho - G4\pi\rho\left(\frac{a^2-r^2}{2}\right.$$
$$= -G\frac{4}{3}\pi\rho\left[\frac{2r^2+3a^2-3r^2}{2}\right]$$
$$V = -GM\left[\frac{3a^2-r^2}{2a^3}\right]\ \left(\because \frac{4}{3}\pi a^3\rho = M\right) \qquad = -G\frac{4}{3}\pi a^3\rho\left[\frac{3a^2-r^2}{2a^3}\right](m$$
$$\therefore\ V = -GM\left[\frac{3a^2-r^2}{2a^3}\right]$$

GRAVITATIONAL FIELD

(i) Point outside the sphere. Potential $V = \frac{-GM}{r}$

$\therefore$ intensity $F = \frac{-dV}{dr} = \frac{-d}{dr}\left[\frac{-G,M}{r}\right] = \frac{-G,M}{r^2}, -$

$\cdots (i)$

(ii) Point on the surface of the sphere. For a point on the surface of the solid sphere, $r = a$, and therefore $F = \frac{-G\cdot M}{a^2}$ [putting $r = a$, in $(t)$]

(iii) Point inside the sphere. Potential at a point inside the solid sphere at a distance $r$ from the centre $O$,

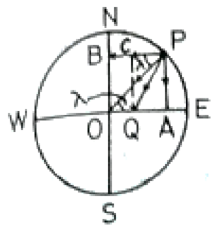
$$V = -G\cdot M\left[\frac{3a^2-r^2}{2a^3}\right]$$

$$\therefore \left.\begin{array}{l}\text{Intensity of the}\\ \text{field at } P\end{array}\right\} = F = -\frac{dV}{dr} = -\frac{d}{dr}\left[-GM\left(\frac{3a^2-r^2}{2a^3}\right)\right]$$
$$= -\frac{GM}{a^3}r$$

Thus, the intensity. of the gravitational field at a point inside a solid sphere is directly proportional to the distance of the point from the centre of the sphere.

Variation of $g$ with Iatitude or rotation of the earth

Let us assume that the earth is a uniform sphere of radius $R$ revolving about its polar diameter $NS$ (Fig. 6.5). Consider a particle of mass $m$ on the surface of the earth at a latitude $\lambda$. If the earth were at rest, a particle of mass $m$ placed at $P$ will experience a force $mg$ along the radius $PO$ towards $O$. Fig. 6.5




Let $\omega$ be the angular velocity of the earth. As the earth revolves, the particle at $P$ will execute circular motion with $B$ as centre and $BP$ as radius. A centrifugal force will develop and the centrifugal force acting on $P$ along $BP$, away from $B = mBP.\omega^2$.

$$= m(R\cos\lambda)\omega^2 \; (\because BP = R\cos\lambda)$$
$$= mR\omega^2\cos\lambda.$$

Force $mg$ acts along $PO$. Resolve $mg$ into two rectangular components (i) $mg\sin\lambda$ along $PA$ and (ii) $mg\cos\lambda$ along $PB$. Out of the resolved component along $PB$, a portion $mR\omega^2\cos\lambda$ is used in overcoming centrifugal force.

Let the net force be represented by $PC$. Then

$PC = mg\cos\lambda - mR\omega^2\cos\lambda$ and $PA = mg\sin\lambda$.

The resultant force $(mg')$ experienced by $P$ is along $PQ$, such that

$$(PQ)^2 = (PC)^2 + (PA)^2 \text{ or } PQ = [(PC)^2 + (PA)^2]^{1/2}$$

$$(PQ)^2 = (PC)^2 + (PA)^2 \text{ or } PQ = [(PC)^2 + (PA)^2]^{1/2}$$
$$mg' = [(mg\cos\lambda - mR\omega^2\cos\lambda)^2 + (mg\sin\lambda)^2]^{1/2}$$

ie. $mg' = [(mg\cos\lambda - mR\omega^2\cos\lambda)^2 + (mg\sin\lambda)^2]^{1/2}$

$$= mg\left[1 + \frac{R^2\omega^4}{g^2}\cos^2\lambda - \frac{2R\omega^2}{g}\cos^2\lambda\right]^{\frac{1}{2}}$$

$\therefore\; mg' = mg\left[1 - \frac{2R\omega^2}{g}\cos^2\lambda\right]^{\frac{1}{2}}$ neglecting $\frac{R^2\omega^4}{g^2}\cos^2\lambda)$

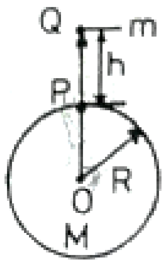
$$= mg\left[1 - \frac{R\omega^2\cos^2\lambda}{g}\right]$$

($\because R\omega^2/g$ is small, its higher powers can be neglected )

$$\therefore\; g' = g\left[1 - \frac{R\omega^2\cos^2\lambda}{g}\right]$$

**Variation of with g altitude**

Let $P$ be a point on the surface of the earth and $Q$ snother point at an altitude $h$ (Fig. 6.6.). Mass of the carth is $M$ and radius of the earth is $R$. Let $g$ be the acceleration due to gravity on the surface of the




earth. Then

The force experienced by a body of mass $m$ at $P$ $\Big\} = mg = \frac{GMm}{R^2}$ ... $(i)$ a body of mass $m$ at $Q$ here $g'$ is the acceleration due to gravity at an altitude $h$.

where $g'$ is the acceleration due to gravity at an altitude $h$. Dividing (ii) by (ii).

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2[1+(h/R)]^2} = \left(1+\frac{h}{R}\right)^{-2}$$

$$= \left(1-\frac{2h}{R}\right) \text{ [neglecting higher powers of } h/R \text{ )}$$

$$\text{or } g' = g\left(1-\frac{2h}{R}\right)$$

Dividing (ii) by (i),

$$\frac{g'}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{R^2[1+(h/R)]^2} = \left(1+\frac{h}{R}\right)^{-2}$$

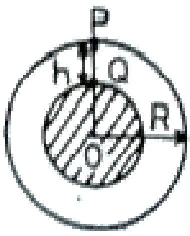
$$= \left(1-\frac{2h}{R}\right) \text{ [neglecting higher powers of } h/R \text{ )}$$

$$\text{or } g' = g\left(1-\frac{2h}{R}\right).$$

This shows that the acceleration due to gravity decreases with increase in altitude.

**Variation of g with depth**

Let $g$ and $g'$ be the values of acceleration due to gravity at $P$ and $Q$ respectively (Fig. 6.7). At $r$, the whole mass of the earth attracts the body.




hic $\therefore\ mg = \frac{GMm}{R^2}$ ... (1)

where $m$ = mass of the body,

$M =$ mass of the earth and

$R =$ Radius of the earth

At $Q$, the body is attracted by the mass of the earth Fig. 6.7 of radius $(R-h)$,

$$\therefore\ mg' = \frac{GM'm}{(R-h)^2}$$

Here, $M = \frac{4}{3}\pi R^3 \rho$ and $M' = \frac{4}{3}\pi (R-h)^3 \rho$ where $\rho$ is the mean density of the earth.

Dividing (2) by (1), $\frac{g'}{g} = \frac{M'}{M}\frac{R^2}{(R-h)^2}$

$$= \frac{\frac{4}{3}\pi (R-h)^3 \rho}{\frac{4}{3}\pi R^3 \rho} \times \frac{R^2}{(R-h)^2} = \frac{(R-h)}{R} = \left(1 - \frac{h}{R}\right)$$

$$\therefore \quad g' = g\left(1 - \frac{h}{R}\right)$$

Therefore, the acceleration due to gravity decreases with increase of depth.

**Summary**

- Dynamics, branch of physical science and subdivision of mechanics that is concerned with the motion of material objects in relation to the physical factors that affect them: force, mass, momentum, and energy.
- According to Newton's law of gravitation, every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

## Sample Question

**2 Marks**

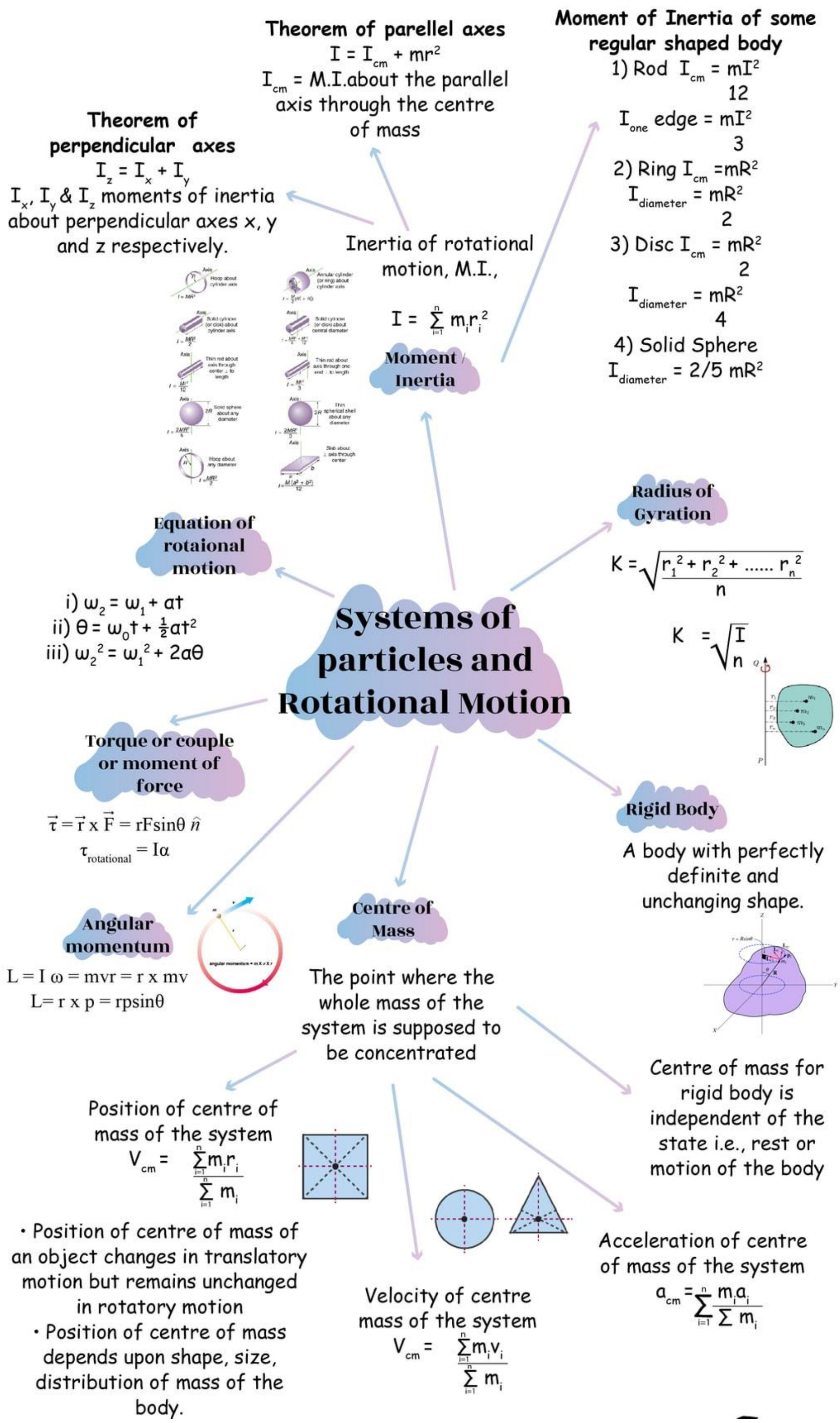
1. Define range of projectiles.
2. State Newton law of gravitation.
3. Define escape velocity.
4. Find and explain the expressions for gravitational field strength and gravitational potential.
5. What is geostationary orbit.
6. What is time of flight.
7. What do you mean by orbital velocity?
8. Define Gravitational potential.
9. State theorem of parallel axis.
10. Define centre of oscillations and centre of suspension.

**5 Marks and 10 Marks**

1. State and explain Kepler’s laws.
2. Write on variation of ‘G’ with height, depth and latitude from the surface of earth.
3. With a diagram explain Boy’s method of finding ‘G’ and give its advantages
4. Define orbital velocity and find its expression for a satellite.
5. Define newton law from kepler’s law planetary motion.
6. Show that the acceleration due to gravity decrease with increase in light.
7. State and explain Kepler’s law of planetary motion.
8. Explain the Newton law of gravitation.
9. Derive an expression for orbital velocity.
10. For a projectile find the time of flight and range
11. Obtain the expresiion for the gravitational potential and the field due to solid sphere
12. Estimate the condition for minimum period of compound pendulum.

# Unit - III

## Mind Map



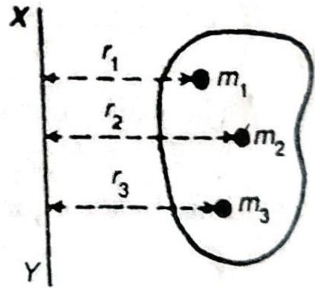
**Objectives**

Learn the basic terms and concepts of rigid body dynamics

## 3.1 Moment of Inertia

### 3.1.1 Introduction

**Definition:** If a rigid body consists of a finite number of particles of masses $m_1$, $m_2$, $m_3$ etc., at distances $r_1$, $r_2$, $r_3$ etc., from a given straight line XY in Fig, the moment of inertia of the body about the given line is given by




$I = m_1 r_1^2 + m_2r_2^2 + m_3^2 + \ldots\ldots\ldots = \Sigma\, mr^2$

Unit: kg $m^2$;

Dimensional formula: $[I] = [ML^2]$.

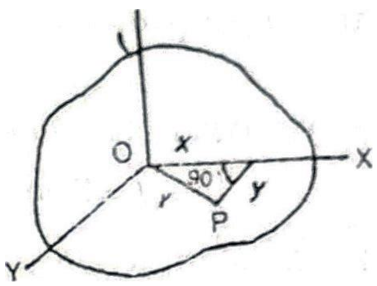
**Radius of Gyration:** Suppose the whole mass of the body (M) is concentrated at a point distant k from the axis such that $M\,k^2 = \Sigma\, mr^2 = 1$. Then k is called the radius of gyration of the body about the given axis. $k = \sqrt{1/M}$.

**Physical Significance of M.I:** In translational motion F = ma.

In rotational motion $\underset{\tau}{\rightarrow} = I\alpha$. This suggests that just as we associate a force with the linear acceleration of a body, so we may associate a torque with the angular acceleration of a body about a given axis. Mass M is a measure of the resistance a body offers to having its translational motion changed by a given force. Similarly, moment of inertia [I] is a measure of the resistance a body offers to having its rotational motion changed by a given torque. Thus M.I plays the same role in rotational motion as mass does in translational motion.

## 3.2 Perpendicular Axes Theorem

**Statement :** If $I_x$ and $I_y$ are the moments of inertia of a lamina about two rectangular axes $OX$ and $OY$ in its plane, its moment of inertia about an axis $OZ$, perpendicular to its plane, is $I_z = I_x + I_y$.




**Proof:** Let OX, OY be the two perpendicular axes in the plane of the lamina and OZ an axis perpendicular to the lamina in Fig. Consider a particle P, of mass m in the plane of the lamina. x,y and r are the distances of the particle from OY, OX and OZ respectively.

Moment of inertia of the particle about $OZ = mr^2$

$\therefore MI$ of the lamina about $OZ = mr^2$

Similarly, $MI$ of the lamina about $OX \;= \Sigma mr^2$

and $MI$. of the lamina about $OY = \Sigma my^2$

But $r^2 = x^2 + y^2$

$$\therefore \Sigma mr^2 = \Sigma m(x^2 + y^2) = \Sigma mx^2 + \sum my^2$$

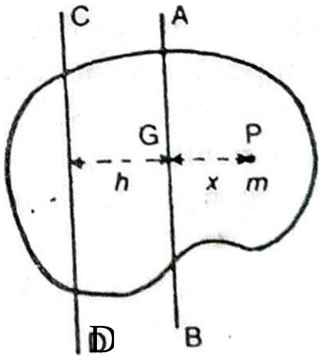
$$\therefore \sum mr^2 = I_z; \sum mx^2 = I_y; \sum my^2 = I_x.$$

$$\therefore I_z = I_x + I_y.$$

**3.3 Theorem of Parallel Axes**

**Statement:** If I is the moment of inertia of a body about an axis through its centre of mass and I' its moment of ineria about a parallel axis at a perpendicular distance $h$ from the first axis at a perpendicular distance h from the first axis, then I' = I + $Mh^2$, where M is the mass of the body.

**Proof:** AB is an axis passing through the centre of mass of the body G in Fig. CD is a parallel axis at a perpendicular distance h from AB. M is the mass of the body.




Consider a particle P of mass m at a distance x from AB.

M.I of the particle P about AB = $mx^2$

M.I of the whole body about AB = I = Σ $mx^2$

M.I. of the particle $P$ about $CD = m(x + h)^2 = m(x^2 + h^2 + 2xh)$

$$= mx^2 + mh^2 + 2mxh$$

MI of the whole body about CD $= I' = \Sigma mx^2 + \sum mh^2 + \sum 2mxh$

$$\text{But} \sum mL^2 \quad = I; \Sigma mh^2 = Mh^2$$

$$I' \quad = I + \text{sill}^2 + 2l_l \sum mx$$

Now, Σ mx is the algebraic sum of the moments of all the particles about G. Since, the body is balanced about the centre of mass G, Σ mx = 0

$$\therefore I' = I + Mh^2$$

**3.4 Moment of Inertia of a Thin Circular Ring**

**(a) About an axis through its centre and perpendicular to its plane.**

Let M be the mass and R the radius of the ring in Fig. Consider a particle of mass m of the ring. Its moment of inertia about an axis through O and perpendicular to the plane of the ring is $m.R^2$.

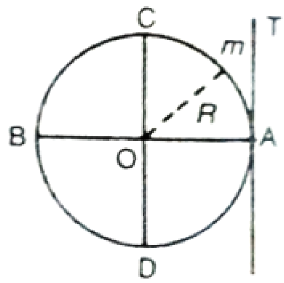
Therefore, the moment of inertia I of the entire ring about the given axis is

$$I = \Sigma\, m\, R^2 = R^2\, \Sigma\, m$$

Or $\mathbf{I = MR^2}$.

**(b) About a Diameter**

By symmetry, the moment of inertia of the ring about any one diameter $(AB)$ is the same as about any other $(CD)$. Let it be $I_\mathrm{d}$. By the theorem of perpendicular axes, the moment of inertia about the central axis, I, will be equal to the sum of its moments of inertia about two mutually perpendicular diameters lying in its plane. Thus,




$$I = I_d + I_d = MR^2$$

$$\therefore I_\mathrm{d} = \frac{1}{2}MR^2$$

**(c) About a Tangent.**

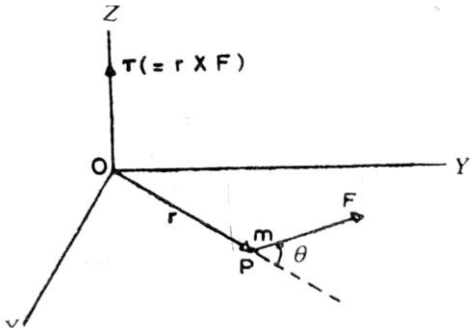
By the theorem of parallel axes, the moment of inertia about a tangent is

$$I_\mathrm{t} = I_\mathrm{d} + MR^2 = \frac{3}{2}MR^2$$

### 3.5 Torque

Consider a particle at a point P in Fig. Let the position vector of P relative to an origin O be r. Let F be the force acting on the particle. Then the torque $\underset{\tau}{\rightarrow}$ acting on the particle with respect to the origin O is defined as

$$\underset{\tau}{\rightarrow} = r \times F \quad \text{--------------------------(1)}$$




Torque is a vector quantity. Its magnitude is given by

$$\tau = rF \sin\theta \quad \text{--------------------------(2)}$$

Here θ is the angle between r and F. Its direction is normal to the plane formed by r and F.

The unit of torque is newton metre (Nm). Its dimensions are $ML^2T^{-2}$.

### 3.6 Angular momentum

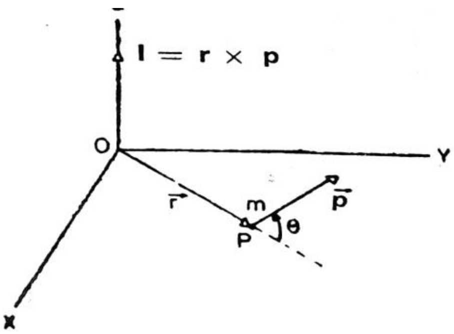
Consider a particle of mass m and linear momentum p at a postion r relative to the origin O in fig. The angular momentum I of the particle with respect to the origin O is defined as

$$\mathbf{I = r \times p = m\,(r \times v).}$$

Angular momentum is a vector. Its magnitude is given by

$$l = r\,p \sin\theta.$$

Here θ is the angle between r and p. Its direction is normal to the plane formed by r and p. The sense is given by the right – hand rule.




The unit of angular momentum is kg $m^2$ $s^{-1}$ or Js.

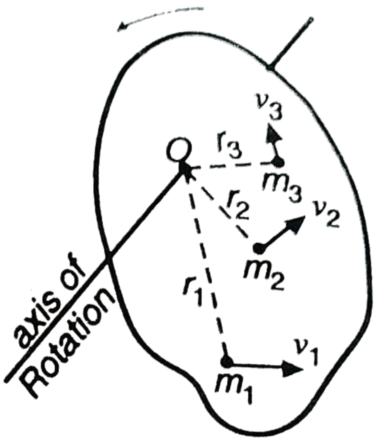
The dimensions of angular momentum are $M L^2T^{-1}$.

For circular motion, v = r ω. The magnitude of I is $mr^2 \omega = I\omega$.

### 3.7 Expression for Torque in Rotational Motion

Definition. *The torque acting on a rotating body about an axis is equal to the sum of the moments of all the forces about the same axis acting on all the particles constituting the body.*

Consider a rigid body rotating with an angular velocity ω about an axis through O (Fig.). The body is composed of a large number of particles of masses $m_1$, $m_2$, $m_3$ .....situated at distances $r_1$, $r_2$, $r_3$........ from the axis of rotation. Every particle revolves with the same angular velocity ω.




The linear velocity ($v_1$) of the particle of mass $m_1$ is $r_1$ ω.

Acceleration of the particle of mass $m_1$ is

$$\frac{dv_1}{dt} = \frac{d}{dt}(r_1\omega) = r_1\frac{d\omega}{dt}.$$

The force acting on the particle of mass $m_1 = m_1 r_1 \frac{d\omega}{dt}$

The moment of this force about the axis of rotation is

$$\left(m_1 r_1 \frac{d\omega}{dt}\right) \times r_1 = m_1 r_1^2 \frac{d\omega}{dt}.$$

Similarly, the moments of the other forces on the other particles about the axis of rotation are

$$m_2 r_2^2 \frac{d\omega}{dt}, m_3 r_3^2 \frac{d\omega}{dt}, m_4 r_4^2 \frac{d\omega}{dt} \ldots\ldots\ldots\ldots$$

$$The\ torque\ acting\ on\ the\ body = Sum\ of\ the\ moments\ of\ all\ the\ forces.$$

$$\begin{aligned}\tau \quad &= m_1 r_1^2 \frac{d\omega}{dt} + m_2 r_2^2 \frac{d\omega}{dt} + m_3 r_3^2 \frac{d\omega}{dt} \ldots.. \\ &= [m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 \ldots]\frac{d\omega}{dt} \\ &= [\Sigma m r^2]\frac{d\omega}{dt}.\end{aligned}$$

$$\Sigma m r^2 = I, the\ moment\ of\ inertia\ of\ the\ body\ about\ the\ axis\ of\ rotation.$$

$$\frac{d\omega}{dt} = \alpha, \text{ the angular acceleration.}$$

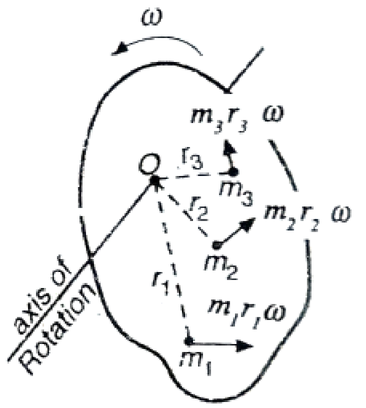
$$\therefore \tau = I\alpha.$$

Torque = moment of inertia × angular acceleration.

### 3.8 Expression for Angular Momentum of a Rotating Rigid Body

**Definition:** *The angular momentum of a rotating body about an axis of rotation is the sum of the moments of linear momenta of all the particles constituting the body.*

Consider a rigid body rotating with an angular velocity ω about an axis through O (Fig.). The body is composed of a large number of particles of masses $m_l$, $m_2$, $m_3$ .....situated at distances $r_1$, $r_2$, $r_3$……… from the axis of rotation.




Every particle revolves with the same angular velocity ω.

The linear velocity ($v_I$) of the particle of mass $m_1$ is $r_1$ ω.

Linear momentum of the particle of mass $m_l$ is $m_1r_1$ ω

The moment of this momentum about the axis of rotation is

$$(m_1r_1\ \omega) \times r_1 = m_1r_1^2\ \omega.$$

Similarly, the moments of the momentum of other particles are

$$m_2r_2^2\ \omega, m_2r_2^2\ \omega, m_4r_4^2\omega\ \ldots\ldots\ldots\ldots$$

The angular momentum L of the rotating body is

$$L = m_1r_1^2\ \omega + m_2r_2^2\ \omega + m_2r_2^2\ \omega\ \ldots\ldots..$$

$$= [\Sigma mr^2]\ \omega.$$

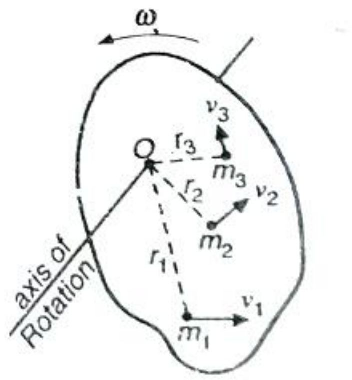
$\Sigma mr^2$ = I, the moment of inertia of the body about the axis of rotation.

$$\therefore L = I\ \omega$$

*Angular momentum = Moment of Inertia × Angular velocity*

### 3.9 Kinetic Energy of Rotation

Consider a rigid body rotating with an angular velocity ω about an axis through O (Fig.). The body is composed of a large number of particles of masses $m_l$, $m_2$, $m_3$ .....situated at distances $r_1$, $r_2$, $r_3$……… from the axis of rotation.




Every particle revolves with the same angular velocity ω.

The linear velocity ($v_I$) of the particle of mass $m_1$ is $r_1$ ω.

K.E of the particle of mass $m_1$ is

$$\frac{1}{2}\, m_1 r_1^2 = \frac{1}{2}\, m_1 r_1^2\, \omega^2.$$

Similarly, K.E of particle of mass $m_2 = \frac{1}{2}\, m_2 r_2^2\, \omega^2$ and so on.

$\therefore$ K.E of the whole body $= \frac{1}{2}\, m_1 r_1^2\, \omega^2 + \frac{1}{2}\, m_2 r_2^2\, \omega^2 + \frac{1}{2}\, m_3 r_3^2\, \omega^2 + \ldots..$

$$= \frac{1}{2}\, \omega^2\, \Sigma m r^2 = \frac{1}{2}\, I\, \omega^2$$

**4.0 Conservation of Linear Momentum**

**Linear momentum:** The linear momentum of a particle is defined as the product of its mass and velocity.

When a particle of mass m is moving with velocity v, its linear momentum p is given by

$$P = mv$$

It is a vector quantity. Its units is *kg m s*$^{-1}$ and dimensions are [$MLT^{-1}$].

If the external force applied to a particle is zero, we have

$$F = dp/dt = d\,(mv)\,/\,dt = 0$$

$$P = mv = \text{a constant.}$$

i.e., *in the absence of an external force, the linear momentum of the particle remains constant.*

This is known as the *law of conservation of linear momentum.*

**4.1 Conservation of Angular Momentum**

$$\overrightarrow{\tau_{ext}} = \frac{d\vec{L}}{dt}.$$

Suppose there is no external torques acting on a body. $\overrightarrow{\tau_{ext}} = o$. Then dL/dt = 0 or L = a constant.

This is the principle of conservation of angular momentum and may be stated thus:

When the *resultant external torque acting on a system is zero, the total vector angular momentum of the system remains constant.*

**Some examples of conservation of angular momentum**

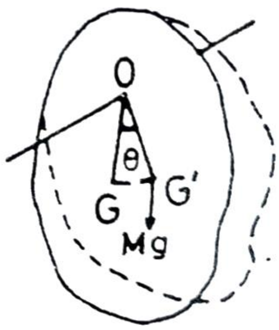
1. A skater or a ballet dancer can increase or decrease angular speed of a spin about a vertical axis. Since $I_{\omega}$ is a constant, when he pulls his arms and legs in 'I' decreases and ω increases, and hence he can spin faster. Similarly, when he extends his arms and legs as far out as possible, *I* increase and ω decreases and hence he can spin slower. A diver also uses the same principle.
2. To conserve its angular momentum, a planet moves faster at its point of nearest approach to the sun than at the point of its farthest approach.

## 4.2 The Compound Pendulum

Any rigid body capable of oscillating freely about a horizontal axis passing through it is a compound pendulum.

**To find the period of oscillation of a compound pendulum:**

Let O be the point of suspension and G the centre of mass (Fig. 3. 5). In the equilibrium position, OG is vertical. OG = h. Suppose the body is given a small angular displacement about O and let go. The centre of mass G is displaced to G' The body oscillates about the equilibrium position. It can be shown that the motion is simple harmonic.




Let M be the mass of the pendulum. The restoring couple due to gravity =Mgh sin θ. The couple is also equal to I ($d^2\theta/dt^2$) where I =M.I of the body about the axis of rotation and ($d^2\theta/dt^2$) = angular acceleration.

The equation of motion of the body is

$$I\,\frac{d^2\theta}{dt^2} = -Mgh\sin\theta$$

Or

$$I\,\frac{d^2\theta}{dt^2} = -\,\frac{Mgh}{I}\sin\theta = -\,\frac{Mgh}{I}\,\theta$$

[∵ sin θ = θ when θ is small]

Now Mgh/I is a constant quantity. Therefore, angular acceleration ($d^2\theta/dt^2$) is proportional to the angular displacement θ and the motion of the body is simple harmonic.

$$\text{Period } T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{\theta}{\left(\frac{Mgh}{I}\theta\right)}}$$

$$= 2\pi\sqrt{\frac{I}{Mgh}}$$

$I = I_{\text{cm}} + Mh^2$ by the parallel axes theorem.

$I = Mk^2 + Mh^2$

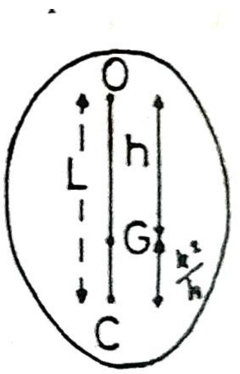
[where $k =$ Radius of gyration of the body about an axis passing through cm ]

or $I = M(k^2 + h^2)$.

$$\therefore T = 2\pi\sqrt{\frac{M(k^2+h^2)}{Mgh}} \text{ or } \mathrm{T} = 2\pi\sqrt{\frac{k^2+h^2}{gh}} ------------ (1)$$

**Equivalent simple pendulum**

A simple pendulum Which oscillates with the same period as the compound pendulum is called the equivalent simple pendulum.




In the case of a simple pendulum of length L, the period

$$\mathrm{T} = 2\pi\sqrt{\frac{L}{g}} \quad \text{------------------(2)}$$

Comparing expressions (1) and (2), the length of the equivalent simple pendulum

$$= \mathrm{L} = \frac{k^2+h^2}{h}$$

**Centre of oscillation:** Produce the line OG to C such that

$$\mathrm{OC} = \frac{k^2+h^2}{h} = \mathrm{h} + \frac{k^2}{h}$$

Then the point C is called the centre of oscillation (Fig)

**To show that he centre of suspension and the centre of oscillation are interchangeable.**

O is the centre of suspension and C is the centre of oscillation of a compound pendulum and OG =h

$$\text{Then OC} = L = \frac{k^2 + h^2}{h}$$

$$\therefore \quad k^2 = hL - h^2 = h(L - h) = OG \times CG.$$

The symmetry of the expression $k^2 = OG \times CG$ shows that, if the body is suspended about a parallel axis through C, O would be the centre of oscillation and suspension are interchangeable.

**Minimum time of oscillation of a compound pendulum**

$$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}$$

$$T \text{ will be a minimum when, } \frac{d}{dh}\left[\frac{k^2+h^2}{h}\right] = 0$$

$$\text{i.e., } \frac{d}{dh}\left[h + \frac{k^2}{h}\right] = 0, \text{ i.e., } \frac{1}{1} - \frac{k^2}{h^2} = 0 \text{ or } k = h.$$

Therefore, the period is a minimum when the radius of gyration about a parallel axis through the centre of mass of the body = The depth of the cm below the point of suspension.

**Determination of g with compound pendulum**

A compound pendulum consists of a heavy uniform metal bar about a metre long. It has a number of holes drilled at regular A compound pendulum consists of a heavy uniform metal bar about a metre long. It has a number of holes drilled at regular

intervals on either side of the centre of mass G


The horizontal knife-edge is passed through the hole near the end A. The period of oscillation is determined and the distance of the knife-edge from the end A is measured. The experiment is repeated and the bar is made to oscillate about the knife-edge placed successively in the different holes from A to B. In each case the period of oscillation (T) and the distance of the position of the knife-edge from the same end A are noted.

A graph is plotted between the period, (on y-axis) and the distance from A (on the x-axis).

Two curves as shown in Fig are obtained. A horizontal line PQRS is drawn cutting both the curves at points P, Q, R and S. P, Q, R and S are then the four points on the bar collinear with the centre of mass having the same period.

PR = QS = L, the length of the equivalent simple pendulum. Therefore, if t be its time period given by the ordinate of any one of the points P, Q, R or S we have,

$$t = 2\pi\sqrt{\frac{L}{g}} \quad \text{or} \quad g = 4\pi^2\frac{L}{t^2}.$$

Knowing L and t, we can calculate the value of g at the given place.

**Kater's Pendulum:**

A Kater's pendulum consists of a rigid metal bar having two knife edges whose positions can be altered along the length of the bar. $M_1$ and $M_2$ are two cylinders of the same size and shape. $M_1$ is made of wood and $M_2$ is made of metal say brass or iron. There is a third adjustable cylinder $M_3$ whose position can also be altered along the length of the bar. On the outer surface of $M_3$ there is an adjustable screw whose position on the surface of $M_3$ can be changed. This helps in the final adjustment of the time period.

The positions of the knife edges $K_1$ and $K_2$ are fixed so that they face each other and are on either side of the centre of gravity of the pendulum. The time period of the pendulum about each knife edge is determined. In the beginning, the difference in the time periods may be large. The position of the mass $M_2$ is adjusted suitably so that the difference in time periods decreases. Finally, by adjusting the positions of the masses $M_3$ and $M_1$, the difference in time periods is made extremely small (say 0.01 s).

Let $l_1$ *and* $l_2$ be the distances of the knife edges from the centre of gravity of the pendulum in the final adjusted positions. Let $t_1$ and $t_2$ be corresponding time periods about the two knife edges $K_1$ and $K_2$.

$$t_1 = 2\pi\sqrt{\frac{k^2+hl_1{}^2}{l_1 h}} \quad \text{---------------------(1)}$$

$$t_2 = 2\pi\sqrt{\frac{k^2+hl_2{}^2}{l_2 h}} \quad \text{---------------------(2)}$$

Squaring (1) and (2)

$$t_1^2 = 4\pi^2\left[\frac{k^2+l_1^2}{l_1 g}\right] \quad \text{----------------------(3)}$$

$$t_2^2 = 4\pi^2\left[\frac{k^2+l_2}{l_2 g}\right] \quad \text{-----------------(4)}$$

From equations (3) and (4)

$$t_1^2 l_1 = \frac{4\pi^2}{g}(k^2+l_1^2) \text{-------------------(5)}$$

and $t_2^2 l_2 = \frac{4\pi^2}{g}(k^2 + l_2^2)$ ---------------------(6)

Subtracting (6) and (5)

$$t_1^2 l_1 - t_2^2 l_2 \quad = \frac{4\pi^2}{g}(k^2 + l_1^2) - \frac{4\pi^2}{g}(k^2 + l_2^2)$$

$$= \frac{4\pi^2}{g}[\cancel{k^2} + l_1^2 - \cancel{k^2} - l_2^2]$$

$$t_1^2 l_1 - t_2^2 l_2 = \frac{4\pi^2}{g}[l_1^2 - l_2^2]$$

$$\frac{4\pi^2}{g} = \frac{t_1^2 l_1 - t_2^2 l_2}{l_1^2 - l_2^2}$$

Using the method of partial fractions

$$\frac{4\pi^2}{g} = \frac{t_1^2 l_1 - t_2^2 l_2}{(l_1^2 - l_2^2)}$$

$$\frac{4\pi^2}{g} \quad = \frac{t_1^2 l_1 - t_2^2 l_2}{(l_1 + l_2)(l_1 - l_2)} = \frac{A}{(l_1 + l_2)} + \frac{B}{(l_1 - l_2)} - - - - - - -(7)$$

$$= \frac{A(l_1 - l_2) + B(l_1 + l_2)}{(l_1 + l_2)(l_1 - l_2)} = \frac{l_1 A - l_2 A + l_1 B + l_2 B}{(l_1 + l_2)(l_1 - l_2)}$$

$$\frac{t_1^2 l_1 - t_2^2 l_2}{(l_1^2 - l_2^2)} \quad = \frac{l_1(A + B) - l_2(A - B)}{(l_1^2 - l_2^2)}$$

$$A + B = t_1^2 \ - - - - - - - - - - - - -(8)$$

$$A - B = t_2^2 \ - - - - - - - - - - - \ (9)$$

solve (3) & 9

$$A + By = t_1^2$$

$$A - B = t_2^2$$

$$2A = t_1^2 + t_2^2$$

$$A = \frac{t_1^2 + t_2^2}{2}$$

8abtract (9) from (B)

$$A + B \quad = t_1^2$$

$$A \quad B_2^2$$

$$\Leftrightarrow \quad (t)^2$$

$$2B \quad = t_1^2 - t_2^2$$

$$B \quad = \frac{t_1^2 - t_2^2}{2}$$

$$\frac{4\pi^2}{g} = \frac{t_1^2 + t_2^2}{2(l_1 + l_2)} + \frac{t_1^2 - t_2^2}{2(l_1 - l_2)}$$

$$\frac{4\pi^2}{g} = \frac{1}{2}\left[\frac{t_1^2 + t_2^2}{l_1 + l_2} + \frac{t_1^2 - t_2^2}{l_1 - l_2}\right]$$

$$g = \frac{8\pi^2}{\left(\frac{t_1^2 + t_2^2}{l_1 + l_2}\right) + \left(\frac{t_1^2 - t_2^2}{l_1 - l_2}\right)} \quad ---------(10)$$

As the values of $l_1$, $l_2$, $t_1$ and $t_2$ are known by experiment, the value of g can be determined, provided the position of the centre of gravity is accurately known.

However, as it is difficult to locate the position of the centre of gravity in a Kater's pendulum, the time periods $t_1$ and $t_2$ are adjusted to be very nearly equal so that in equation (10).

$\frac{t_1^2 - t_2^2}{l_1 - l_2}$ is negligibly small.

From equation (s), $g = \frac{8\pi^2}{\frac{t_1^2 + t_2^2}{l_1 + l_2}}$

Taking

$$t_1 = t_2 = t,$$
$$l_1 + l_2 = L$$
$$g = \frac{8\pi^2}{\left(\frac{2t^2}{L}\right)} = \frac{8\pi^2 L}{2t^2}$$
$$g = \frac{4\pi^2 L}{t^2} \quad ---------(11)$$

Here L is the distance between the two knife edges. Equation (9) is similar to the equation of a simple pendulum

**Summary:**

- Rigid body dynamics is the study of the motion in space of one or several bodies in which deformation is neglected.
- The assumption that the bodies are rigid (i.e. they do not deform under the action of applied forces) simplifies analysis, by reducing the parameters that describe the configuration of the system to the translation and rotation of reference frames attached to each body.

**Important Questions**

1. Derive an expression for the moment of inertial of a diatomic molecule.

2. Write a note on Kater's Pendulum.

3. How to determine the acceleration due to gravity using compound pendulum.

4. What is angular momentum?

5. State parallel axis theorem and perpendicular axis theorem.

# Unit – IV Mind Map

# Mechanical Properties of Fluids

## Bernoulli's Principle

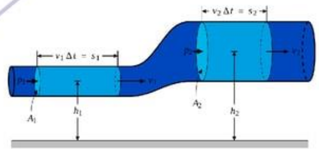
For an incompressible non-viscous streamline, irrotational flow of fluid,
$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

### Application

Device used to measure the rate of flow of liquid.
Volume of liquid flowing per second
$Q = a_1 a_2 \sqrt{\frac{2h\rho_m g}{\rho(a_1^2 - a_2^2)}}$

**Toricell's Law:**
Velocity of efflux of liquid through an orifice
$V = \sqrt{2gh}$

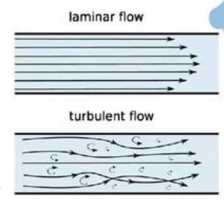
- Lift of an aircraft wing.
- Sprayer or automizer
- Blowing off the roofs during windstorm



## Flow of Fluids

**Streamline:** In liquid flow when the Velocity is less than critical velocity, each particle of the liquid passing through a point travels along the same path and same velocity as the preceding particles.

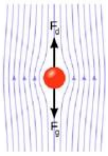
**Turbulent:** When velocity of liquid flow is greater than critical velocity and particles follow zig-zag path.




## Viscocity

Opposing force between different layers of fluid in relative motion
Viscous drag $F = -\eta A \frac{dv}{dx}$
$\eta$ = coefficient of viscocity

Stroke's law $F = 6\eta vr$



Surface tension $S = F/l$

Surface energy $= \frac{\text{work done in increasing area}}{\text{increase in surface area}} = \frac{W}{\Delta A}$

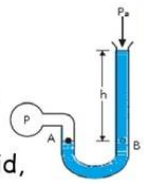
Capillary rise or fall, $h = \frac{2S\cos}{r\rho g}$

Excess Pressure inside a drop (liquid) $P_{excess} = \frac{2S}{R}$

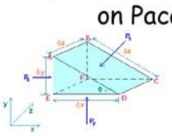
Excess pressure inside the bubble (Soap) $P_{excess} = \frac{4S}{R}$

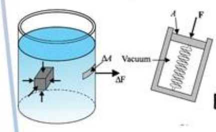
## Laws of Fluids

**Equation of Continuity**
$m = a_1 v_1 \rho_1 = a_2 v_2 \rho_2$
for an incompressible liquid,
$\rho_1 = \rho_2$ then $a_1 v_1 = a_2 v_2$ or
$av = \text{constant}$



**Pascal's law:** The pressure exerted at any point on an enclosed liquid is transmitted equally in all direction.
Hydraulic brakes and hydraulic lifts are based on Pacal's law.





Density$(\rho) = \frac{\text{Mass(m)}}{\text{Volume (v)}}$
Density of water at 4°C i.e, maximum density of water = $1.0 \times 10^3 kg/m^3$

Relative density or specific gravity = $\frac{\text{density of substance}}{\text{desnity of water at 4°C}}$

Pressure(P) = $\frac{\text{thrust(F)}}{\text{area(A)}} = \lim \frac{\Delta E}{\Delta A}$
Pressure exerted by a liquid column of height h, $(\rho) = h\rho g$

**Atmospheric Pressure(Pa)**
Pressure (atm) exerted by the atmosphere.
At sea level, 1 atm=pressure exerted by 0.76m of Hg=hρ
$g = 0.76 \times 13.6 \times 10^3 \times 9.8$
$= 1.013 \times 10^5 \ Nm^{-2} = 101.3 kPa$

**Absolute pressure(P)**
Total or actual pressure at a point.
Absolute pressure= atmospheric pressure + gauge pressure

**Guage Pressure(Pg):**
Difference between the absolute pressure at a point and the atmospheric pressure.
Pg=absolute pressure(P) – atmospheric pressure(Pa)

**Objectives**

Understand the concepts of like center of pressure, Floating bodies and Liquid motion.

**4.1 Pressure and Thrust**

Since a liquid possesses weight, it exerts force on all bodies in contact with it. The ratio between the small force δF and the area δA on which it acts gives the Thus, pressure = δF/δA.

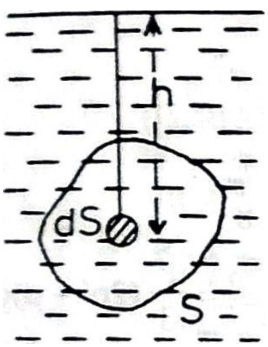
Pressure at a point = $\lim_{\delta A \to 0} \left(\frac{Force}{Area}\right) = \lim_{\delta A \to 0} \left(\frac{\delta F}{\delta A}\right) = \frac{dF}{dA}$.

Unit of pressure is $Nm^{-2}$ . It can be shown that the hydrostatic pressure due to a liquid column of density ρ at a depth h from the surface =hρg .

*The total force exerted by a liquid column on the whole of the area in contact with it is called* *thrust.* Thus , thrust = *pressure x area*. Unit : Newton (N). The thrust is always normal to the plane area.

**4.2 Thrust on a plane surface immersed in a liquid at rest**

Consider a plane lamina of area S immersed in a liquid of density ρ ( Fig. 4.1 ). Divide the area into a very large number of small elements. Let dS be an elementary area at a depth h below the free surface of the liquid.




Thrust on dS at right angles to it = hρg dS

∴ The resultant thrust on the whole surface $= \int h\rho g\, dS = \rho g \int h\, dS$.

Let $\bar{h}$ be the depth of the C.G., from the liquid surface. From the theorem of moments,

$$h_1 dS_1 + h_2 dS_2 + h_3 dS_3 + \ldots\ldots\ldots = \bar{h} S$$

$$\text{or } \int h\, dS = \bar{h} S$$

∴ Resultant thrust on the immersed plane surface = $\bar{h}\rho g$ x S = pressure at C.G. of area x area of the plane.

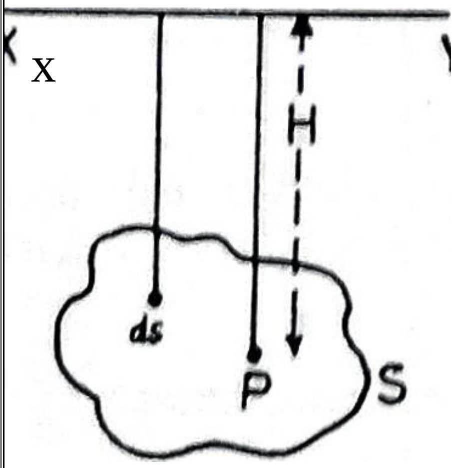
**4.3 Centre of pressure**

We know that the liquid pressure acts normally at every point of the immersed area, The force acting on an elementary area like dS is hρgdS, The thrusts on different elements of the plane form a set of like *parallel forces*, All these parallel forces can be compounded into a resultant acting at some definite point on the plane area, This point is called the centre of pressure.

*The centre of pressure of a plane surface In contact with a fluid is the point on the surface through which the line of action of the resultant of the thrusts on the various elements of the area passes.*

**4.3.1 Determination of Centre of pressure-—General case**:

Consider a plane surface of area S immersed vertically in a liquid of density p. Let XY be the Y surface of the liquid.




Thrust on an elementary area dS at a depth h = h ρgdS

Moment of this thrust about XY = (h ρgdS) × h = $h^2$ ρgdS

Resultant moment of all thrusts = $\int h^2$ ρgdS

Where the integration is carried over all the elements of the plane area.

Resultant thrust on the plane area = $\int$ h ρgdS

Let the centre of' pressure of the plane area be at the point P. Let the distance of P from XY be H,

Moment of the resultant thrust about XY = H $\int$ h ρgdS.

By definition of the resultant of several forces, we get

Moment of the resultant force = resultant of the moments of the forces.

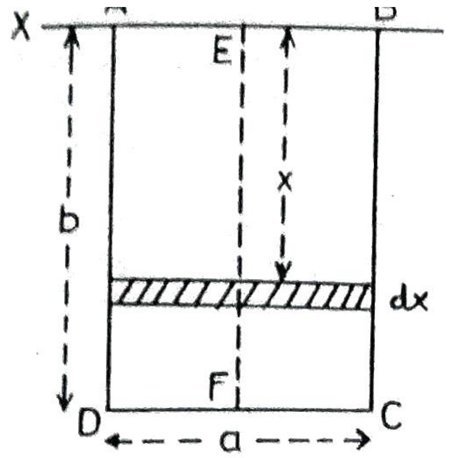
H $\int$ h ρgdS = $\int h^2$ ρgdS

$$H = \frac{\int h^2 dS}{\int h\, dS}$$

The result holds good for any inclined position of the plane also.

**4.4 Centre of pressure of rectangular lamina immersed vertically in a liquid with one edge in the surface of the liquid.**

Let ABCD be a plane rectangular lamina immersed vertically in a liquid of density ρ with one edge AB in the surface XY of the liquid (Fig). Let AB = a and AD b. Divide the rectangle into a number of narrow strips parallel to AB, Consider one such strip of width dx at a depth x below the surface of the liquid.




The thrust acting on the strip = $(x\rho g) \times (a dx) = x\rho ga\, dx$

Moment of this thrust about AB = $(x\rho ga\, dx) \times x = x^2 \rho ga\, dx$

Sum of the moments of the thrusts on all the strips = $\int_0^b x^2 \rho ga\, dx$

Resultant of the thrusts on all the strips = $\int_0^b x \rho ga\, dx$

Moment of the resultant thrust about AB = $H \int_0^b x \rho ga\, dx$

Where H = depth of the Centre of pressure below AB

$$H \int_0^b x \rho ga\, dx = \int_0^b x^2 \rho ga\, dx$$

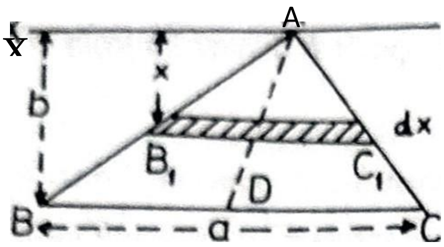
$$H \rho ga \frac{b^2}{2} = \rho ga \frac{b^3}{3}$$

$$H = \frac{2}{3} b.$$

The thrust on every elementary strip act through its midpoint. Hence the centre of pressure will lie on EF where E and F are the mid-points of AB and DC.

**4.5 Centre of pressure of a triangular lamina immersed vertically in a liquid with its vertex in the surface of the liquid and its base horizontal.**

Let ABC be a triangular lamina immersed vertically in a liquid with its vertex A in the surface XY of the liquid and with its base BC horizontal. BC = a. Let the depth of the base of the lamina be b from the free surface of the liquid. Divide the triangle into a number of elementary strips of width dx parallel to the base BC. Consider one strip $B_1 C_1$ of width dx at a depth x below the surface XY.




Area of the strip $B_1C_1 = B_1C_1$ dx = (ax/b)dx

Thrust on the strip $B_1C_1 = (x\rho g) \times (ax/b)dx$

Moment of this thrust about $XY = \left(\frac{ax^3\rho g}{b}dx\right)$

Total moment due to all the strips $= \int_0^b \frac{a\rho g}{b}x^3dx.$

Resultant of the thrusts on all the strips $= \int_0^b \frac{a\rho g}{b}x^2dx.$

Moment of the resultant thrust about $XY = H\int_0^b \frac{a\rho g}{b}x^2dx.$

Here $H =$ the depth of the centre of pressure below $XY$.

Since the two moments are equal,

$$\int_0^b \frac{a\rho g}{b}x^3dx = H\int_0^b \frac{a\rho g}{b}x^2dx.$$

Or

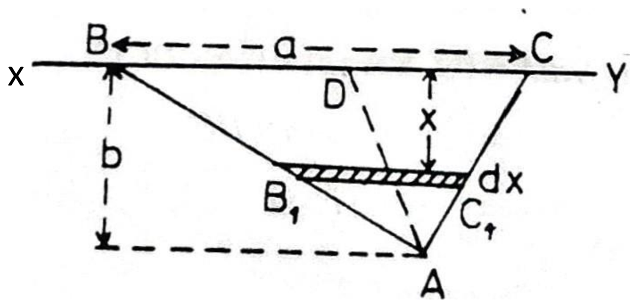
$$\frac{a\rho g}{b}\left[\frac{b^4}{4}\right] = H\,\frac{a\rho g}{b}\left[\frac{b^3}{3}\right]$$

$$H = \frac{3}{4}b.$$

The centre of pressure lies on the line joining the mid-points of the strips. i.e., lies on the median AD at a depth 3b/4 below the surface XY.

**4.6 Centre of pressure of a triangular lamina immersed in liquid with one side in the surface, where one side in the surface, when there is no external pressure**

Let ABC be a triangular lamina immersed in a liquid with its base BC = a in the surface XY of the liquid (Fig). Let AD be a median of the triangle. Let b be the depth of the vertex A below the surface XY. Divide the triangle into a number of elementary strips of width dx parallel to the base BC. Consider one such strip $B_l$ $C_l$ at a depth x below BC.




Area of the strip $B_1C_1 = B_1C_1 dx = \frac{a(b-x)}{b} dx$

$\left(\text{Since } \frac{B_1C_1}{a} = \frac{b-x}{b}\right.$

Thrust on the strip $B_1C_1 = x\rho g \frac{a(b-x)}{b} dx$

Moment of this thrust about $XY = x^2 \rho g \frac{a(b-x)}{b} dx.$

Total moment due to all the strips $= \int_0^b x^2 \rho g \frac{a(b-x)}{b} dx.$

Resultant of the thrusts on all the strips $= \int_0^b x\rho g \frac{a(b-x)}{b} dx.$

Let $H$ be the depth of the centre of pressure below $XY$.

Moment of the resultant thrust about $XY = H \int_0^b x\rho g \frac{a(b-x)}{b} dx.$

$$\therefore \int_0^b x^2 \rho g \frac{a(b-x)}{b} dx = H \int_0^b x\rho g \frac{a(b-x)}{b} dx.$$

Or

$$H \int_0^b x(b-x) dx = \int_0^b x^2 (b-x) dx$$

or

$$H\left[\frac{b^3}{2} - \frac{b^3}{3}\right] = \left[\frac{b^4}{3} - \frac{b^4}{4^*}\right]$$

or

$$H = \frac{b}{2}$$

The centre of pressure is on the median AD at a depth b/2 below XY.

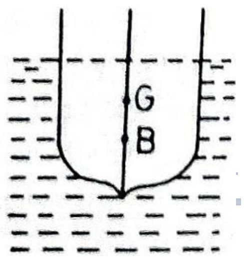
## 4.7 Floating Bodies

### 4.7.1 Laws of Floatation:

1. The weight of the floating body is equal to the weight of the liquid displaced by it.
2. The centre of gravity of the floating body and the centre of gravity of the liquid displaced (i.e., the **centre of buoyancy**) are in the same vertical line.

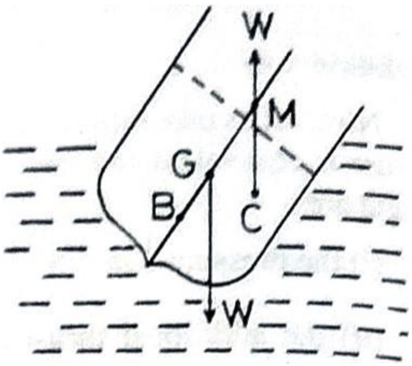
### 4.7.2 Stability of Floating Bodies:

The equilibrium of a freely floating body is said to be stable, if on being slightly displaced, the body returns to the original equilibrium position.

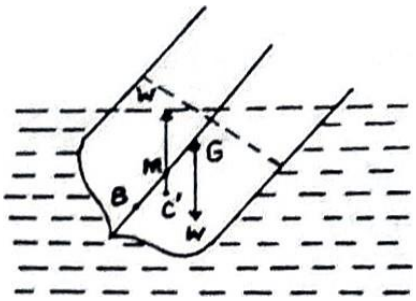



Consider a floating body in equilibrium. G is the centre of gravity of the floating body and B is the centre of buoyancy. The line BG is vertical.

When the floating body is slightly displaced C is the new centre of buoyancy. The vertical line through C meets the original vertical line BG at M. M is called the '*metacentre*' of the floating body. GM is called the ***metacentric height***, The weight of the body W acts vertically downwards through G. The upthrust of value W acts vertically upwards through C. If the metacentre is above G, the couple due to the forces at G and C is anticlockwise and brings the floating body back to its original position, Hence in this case the equilibrium is stable.




But if M lies below G, the couple due to the forces at G and C is clock-wise and the couple tends to turn the body away from the equilibrium position. Hence this equilibrium is unstable. Hence *for a floating body to be in stable equilibrium, the metacentre must be always above the centre of gravity of the body.*

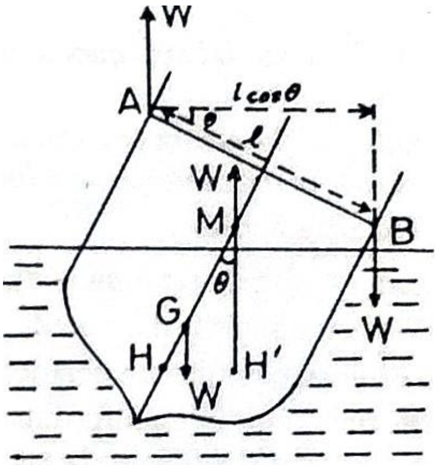



**Note :** In the case of a sphere floating in a liquid, a tilt one way or other does not change the shape of the Fig. displaced liquid. Hence M coincides with G all the time. Therefore, it is said to be in neutral equilibrium and it continues to float in all positions.

### 4.7.3 Experimental determination of the metacentric height of a ship.

The weight of the ship W is determined by the displacement method. Two identical boats are attached one on each side of the ship. In Fig.

A and B represent the boats at a distance $l$ apart on the deck. Filling A and B alternately with water is equivalent to moving a known weight w from A to B across the deck. Filling the boat B with the same mass of water as in A, turns the ship through an angle θ. The tilt θ is determined by means of a plumb line suspended in the ship.




Now, this shift of weight w from A to B is equivalent to a downward force w at B and an upward force w at A constituting a couple of moment W$l$ cos θ. Let H and H' be the original and altered positions of centre of buoyancy, G the centre of gravity of the ship and GM the metacentric height.

The weight W of the ship acting downwards at G and an equivalent upward thrust at the new centre of buoyancy H' form a couple with an opposing moment W × GM sin θ. For equilibrium in the tilted position of ship,

$$W \times GM \sin\theta = w \times l \cos\theta$$

Or

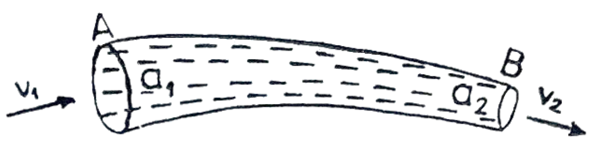
$$GM = \frac{wl}{W \tan\theta}$$

$$GM = \frac{wl}{W\theta} \qquad [\ \theta \text{ being small, } \tan\theta = \theta\ ]..$$

Thus knowing W, w, $l$ and θ. We can easily calculate the metacentric height of the ship.

**4.8 Equation of continuity**

The equation of continuity is an expression of the law of conservation of mass in fluid mechanics. Fig represents a tube of varying cross-section through which a non-viscous incompressible fluid of density p flows. Let al and be the cross- sectional areas of the tube points A and B. Let the velocity of the fluid at A and B be $v_1$ and $v_2$ respectively. Since the fluid is incompressible, in the steady state, mass of fluid entering the tube per second through the section A = mass of fluid leaving the tube per second through the section B.



Mass of fluid entering the tube per second across the section A = $a_1v_1\rho$

Mass of fluid leaving the tube per second across the section B = $a_2v_2\rho$

$a_1v_1\rho = a_2v_2\rho$ or $a_1v_1 = a_2v_2$.

Thus the product av is constant along any given flow tube. It follows that the speed of flow through a tube is inversely proportional to the cross-sectional area of the tube. It means that where the area of cross-section of the tube is large, the velocity is small and vice versa.

**4.9 Energy of the liquid:**

A liquid in motion possesses three types of energy, viz., (i) potential energy (ii) kinetic energy and (iii) pressure energy.

I. **Potential Energy:** If we have a mass m of the liquid at a height h above the earth's surface, its P.E = mgh.

P.E per unit mass of the liquid = gh.

P.E per unit volume of the liquid = ρgh.

II. **Kinetic Energy:**

The K. E. is the energy possessed by the liquid by virtue of its motion. The K. E. of a mass m of a liquid flowing with a velocity v is $\frac{1}{2}\, m\, v^2$

K. E per unit mass of the liquid = $\frac{1}{2}\, v^2$

K. E per unit volume of the liquid = $\frac{1}{2}\, \rho v^2$

III. **Pressure Energy:** Consider an incompressible non-viscous liquid contained in a tank (fig). The tank has a side tube at an axial depth h below the free surface of the liquid in the tank. The side tube is fitted with a smooth piston. Let a be the area of cross-section of the side tube. Let ρ be the density of the liquid. Hydrostatic pressure of the liquid on the piston = p = hρg. Force on the piston = pa.

If the piston is moved inwards through a distance x, the work done =ρax. This will force a mass ρax of the liquid into the tank. This work done on the mass ρax accounts for an expenditure of energy ρax. This is stored up as pressure energy of the same mass without imparting any velocity to it.

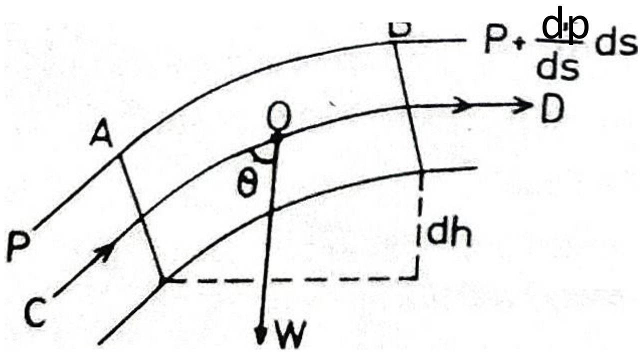
Pressure energy per unit mass = $\frac{pax}{\rho ax} = \frac{P}{\rho}$

Pressure energy per unit volume of the liquid = ρ

The three forms of energy possessed by a liquid under flow are mutually convertible.

**4.9.1 Euler's Equation for Unidirectional flow**

Consider a very small element AB of a flowing liquid (Fio). ρ is the density of the liquid. dA is the area of cross-section of the element. ds is the length of the element.




Volume of the element $= dA \cdot ds$

Mass of the element $= \rho dA \cdot ds$.

Weight of the element $= \rho dA \cdot ds \cdot g$.

Let the streamline $CD$ make an angle $\theta$ with the vertical at $O$.

Component of the weight of the element along the streamline

$= \rho dA \cdot ds \cdot g \cdot \cos\ \theta$.

Let the pressure of the liquid at $A$ be $P$.

Then, the pressure at $B = \left(P + \frac{dP}{ds}ds\right)$

Resultant force on the element $= PdA - \left(P + \frac{dP}{ds}ds\right)dA = -\frac{dP}{ds}dsdA$. This force is directed from $B$ to $A$. The component of the weight of the element is also directed from $B$ to $A$.

The net force acting on the element $= -\frac{dP}{ds}dsdA - \rho dsdAg\cos\ \theta$

Let $dh$ be the difference in heights between $A$ and $B$. Then $\cos\ \theta = dh/ds$.

∴ the resultant force on the element is

$$F = -\frac{dP}{ds}dsdA - \rho dsdAg\frac{dh}{ds} = -\frac{dP}{ds}dsdA - \rho dAdhg$$

Let the velocity of the element of liquid be $v$. Then its acceleration is

$$a = \frac{dv}{dt} = \frac{ds}{dt}\frac{dv}{ds} = v\frac{dv}{ds}$$

According to Newton's II law of motion, $F = ma$.

$-\frac{dP}{ds}dsdA - \rho dAdhg = \rho dAdsv\frac{dv}{ds}$ or $dP + \rho gdh = -\rho vdv$ $\frac{dP}{\rho g} + dh + \frac{vdv}{g} = 0$

or $\frac{dP}{\rho g} + dh + \frac{vdv}{g} = 0$

This equation is called Euler's equation of motion.

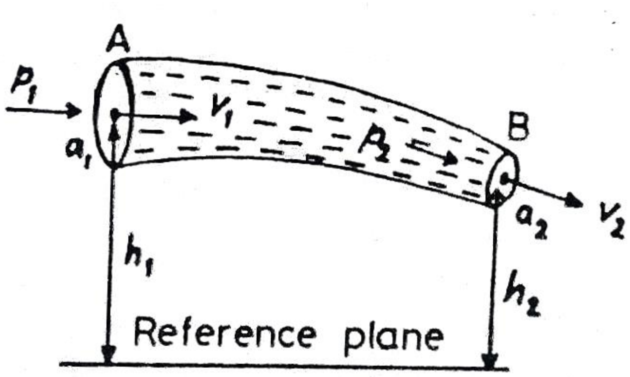
Integrating, $\frac{P}{\rho g} + \frac{v^2}{2g} + h =$ constant.

or $\frac{P}{\rho} + \frac{1}{2}v^2 + gh =$ constant.

This is Bernoulli's equation.

### 4.9.2 Bernoulli's theorem

**Statement.** *The total energy of an incompressible non-viscous fluid flowing from one point to another, without any friction remains constant throughout the motion.*




**Explanation:** According to this theorem, the sum of kinetic, Potential and pressure energies of any element of an incompressible fluid in streamline now remains constants Suppose the height of an element of fluid of density ρ above ground level is h. Let it be moving with a velocity v. Let it have pressure p. Then, its total energy per unit volume is

$$E = \rho v^2 / 2 + \rho\, gh + \rho.$$

Bernoulli's theorem states that E is a constant.

If at two points in the fluid the velocities are $v_1$, $v_2$ the heifhts are $h_1$, $h_2$ and the pressures are $p_1$, $p_2$ then

$$\rho\, v_1^2 / 2 + \rho\, gh_1 + p_1 = \rho\, v_2^2 / 2 + \rho\, gh_2 + p_2.$$

The K. E. per unit weight is called *velocity head* and is equal to $v^2/2g$. The P. E. per unit weight is called the *gravitational head* and is equal to h. The pressure energy per unit weight is called the *pressure head* and is equal to p/ρg. Bernoulli's equation can be written as

$$v^2 / (2g) + h + p/(\rho g) = \text{constant}.$$

*i.e.*, velocity head + gravitational head + pressure head = constant.

In the case of liquid flowing along a *horizontal pipe*, the gravitational head h is a constant.

$$\frac{v^2}{2g} + \frac{p}{\rho g} = constant \text{ or } \frac{v^2}{2} + \frac{p}{\rho} = constant$$

Or $$p + \rho v^2 / 2 = \text{constant}.$$

Or Static pressure + dynamic pressure = constant.

This expression shows that greater velocity corresponds to a decrease in pressure and vice versa. i. e., points of maximum pressure correspond to those of minimum velocity and vice versa. This principle may be used to determine fluid speeds by means of pressure measurements.

**Example.** Venturimeter, Pilot tube, etc.

**Proof.** Consider a fluid in stream line motion along a non-uniform tube(fig). Let A and B be two transverse sections of the tube at heights $h_1$ and $h_2$ from a reference plane (the surface of the earth). Let $a_l$ and $a_2$ be the areas of cross-section at A and B. Let $v_1$ and $v_2$ be the velocities of the fluid at A and B. Let $p_1$ be the pressure at A due to the driving pressure head, Let $p_2$ be the pressure at B. Since $a_1 > a_2$. $v_2 > v_1$. Hence the fluid is accelerated as it flows from to B.

Work done per second on the liquid entering at $A$ is $W_1 =$ Force at $A \times$ Distance moved by the liquid in 1 second

$$= p_1 a_1 \times v_1 = p_1 a_1 v_1$$

Work done per second by the liquid leaving the tube at $B$ is

$$W_2 = p_2 a_2 v_2$$

∴ Net work done by the fluid in passing from $A$ to $B$

$$W = W_1 - W_2 = p_1 a_1 \dot{v}_1 - p_2 a_2 v_2$$

But

$$a_2 v_2 = a_1 v_1$$

∴

$$W = (p_1 - p_2) a_1 v_1$$

The work done on the liquid is used in changing its potential and kinetic energies.

$$\begin{aligned} \text{Decrease in P. E. } &= (a_1 v_1 \rho) g (h_1 - h_2) \\ \text{Increase in K. E. } &= \frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2) \end{aligned}$$

Hence, the total gain in the energy of the system when the liquid flows from $A$ to $B$

$$= \frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2) - (a_1 v_1 \rho g)(h_1 - h_2)$$

$$\therefore (p_1 - p_2)a_1 v_1 = \frac{1}{2}(a_1 v_1 \rho)(v_2^2 - v_1^2) - (a_1 v_1 \rho g)(h_1 - h_2)$$

$$\text{or } p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$\text{or } p + \frac{1}{2}\rho v^2 + h\rho g = \text{ constant}$$

$$\text{or } \frac{p}{\rho} + \frac{1}{2}v^2 + hg = \text{ constant}$$

$$\text{or } \frac{p}{\rho g} + \frac{v^2}{2g} + h = \text{ constant.}$$

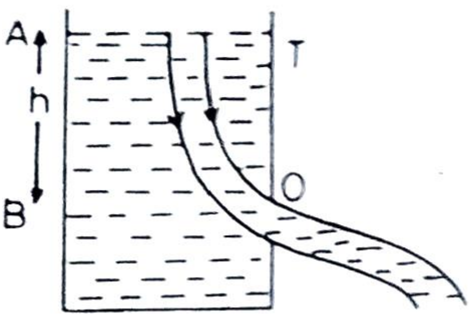
p/(ρg) is called the pressure head. $v^2/(2g)$ is called the velocity head and h is called the gravitational head. Hence, Bernoulli's theorem may be stated as follows. When an incompressible non-viscous fluid flows in stream line motion, the sum of the pressure head, velocity head and gravitational head remains constant throughout its motion.

**Applications of Bernoulli's theorem**

**(i) Torricelli's theorem:**

Consider a vessel T having a liquid of density p. Let the surface of the liquid be at a height h above the level of the orifice O (Fig).

If the vessel is sufficiently wide, the velocity of the liquid on the free surface A may be taken as zero. The pressure on the free surface is atmospheric pressure. Since the pressure is also atmospheric at the orifice, where the liquid emerges, it plays no part in the flow of the liquid. Let v be the velocity of the liquid at the orifice.




Consider a tube of flow beginning at A and ending at O.

Total energy at A = pressure energy + potential energy + kinetic energy

= 0 + hg + 0 = hg.

Total energy at O = 0 + 0 + ½ $v^2$ = ½ $v^2$.

Since total energy remains the same

½ $v^2$ = hg     or     v = $\sqrt{2gh}$

This gives the velocity of efflux at the orifice. This result is known as Torricelli's theorem. It states that the velocity of efflux of a liquid through an arifice is equal to that which a body attains in falling freely from the surface to orifice.The liquid that through the orifice with the velocity v = $\sqrt{2gh}$ flows out along a parabolic path. Let $h_1$ be the height of the orifice above the bottom of the vessel. The liquid reaches the horizontal plane through the bottom of the vessel in time. t = $\sqrt{\frac{2h_1}{g}}$

The escaping liquid will strike the horizontal plane through the bottom of the vessel at a distance H, where H = v × t

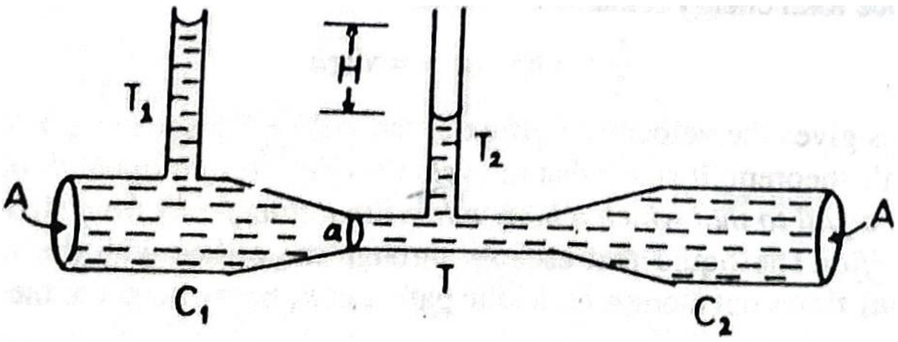
$$H = \sqrt{2gh} \times \sqrt{\frac{2h_1}{g}} = 2\sqrt{hh_1}$$

The range H is maximum for a given height h + $h_1$ if h = $h_1$.

**Vena contracta:** It is to be noted that the volume of the liquid that moves across the orifice in one second cannot be computed by multiplying the area of orifice with the velocity of efflux. The streamlines converge as they approach the orifice. Therefore, the fluid velocities, as the jet leaves the hole, are not parallel to one another but have components inwards towards the centre of the stream. This inward momentum of the emergent fluid causes a contraction of the jet. After the jet has gone a little way, the contraction stops and the velocities become parallel. This point where the contraction of the jet stops and the fluid velocities become parallel is known as *vena contracta*. It is at this point that the velocity multiplied by area gives the rate of flow of the liquid. For an orifice of circular shape, the area of the vena contracta is about 65% of the area of the orifice.

**(ii) Venturimeter**.

It is a device based on Bernoulli's principle. It is used for measuring the rate of flow of liquids in pipes. It consists of two wide conical tubes $C_l$ and $C_2$ with a constriction T between them. Tis called the throat. Let the area of cross-section of $C_l$ and $C_2$ be A. Let a be the area of cross-section of the throat.




When the flow is steady, let V be the volume of water flowing per second through the venturi meter.

Then, $V = Av_1 = av_2$.

Here $v_1$ = velocity in $C_1$ or $C_2$ and $v_2$ = velocity in $T$.

$$v_1 = V/A \text{ and } v_2 = V/a.$$

Hence velocity of water in $T$ is greater than the velocity in $C_1$ and $C_2$. Consequently, the pressure in $T$ is smaller than the pressure in $C_1$ and $C_2$. This difference in pressure $H$ is measured by the difference of the water levels in the vertical glass tubes $T_1$ and $T_2$ connected to $C_1$ and $T$ respectively. Let $p_1$ and $p_2$ be the pressures in the wider limb and throat respectively.

According to Bernoulli's equation for a horizontal flow,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g}$$

$$\text{or } \frac{p_1 - p_2}{\rho g} = \frac{{v_2}^2 - {v_1}^2}{2g}$$

The difference in pressure in $C_1$ and $T = p_1 - p_2 = H\rho g$.

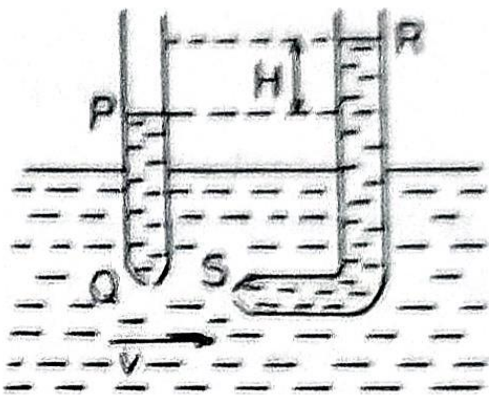
Hence

$$\frac{H\rho g}{\rho g} = \frac{1}{2g}\left[\frac{V^2}{a^2} - \frac{V^2}{A^2}\right] = \frac{V^2}{2g}\left[\frac{A^2 - a^2}{A^2 a^2}\right]$$

$$\therefore V = Aa\sqrt{\frac{2gH}{(A^2 - a^2)}}$$

The rate of flow of water through the pipeline can he determined by measuring $H$, and knowing the constants $A, a$, and $g$.

**(iii)Pitot tube:** It is an instrument used to measure the rate of flow of water through a pipe-line. It is based on Bernoulli's principle. It consists of two vertical tubes PQ and RS with small apertures at their lower ends (Fig). The plane of aperture of the tube PQ is parallel to the direction of flow of water. The plane of aperture of the tube RS faces the flow of water perpendicularly. The rise of the water column in the tube RS therefore, measures the pressure at S.




Let $p_1$ and $p_2$ be the pressures of water at Q and S respectively. Let v be the velocity of water at Q. Since the water is stopped in the plane of the aperture S of the tube RS, its velocity at S becomes zero. Let be difference of level in the two tubes. Applying Bernoulli's theorem to the ends Q and S.

$$\therefore \ \frac{1}{2}v^2 + \frac{p_1}{\rho} = \frac{p_2}{\rho} \text{ or } v^2 = \frac{2}{\rho}(p_2 - p_1) = \frac{2}{\rho}H\rho g$$

$$\therefore \ v = \sqrt{2gH}.$$

Rate of flow of water $= av$

where $a =$ area of cross-section of the pipe.

**Summary**

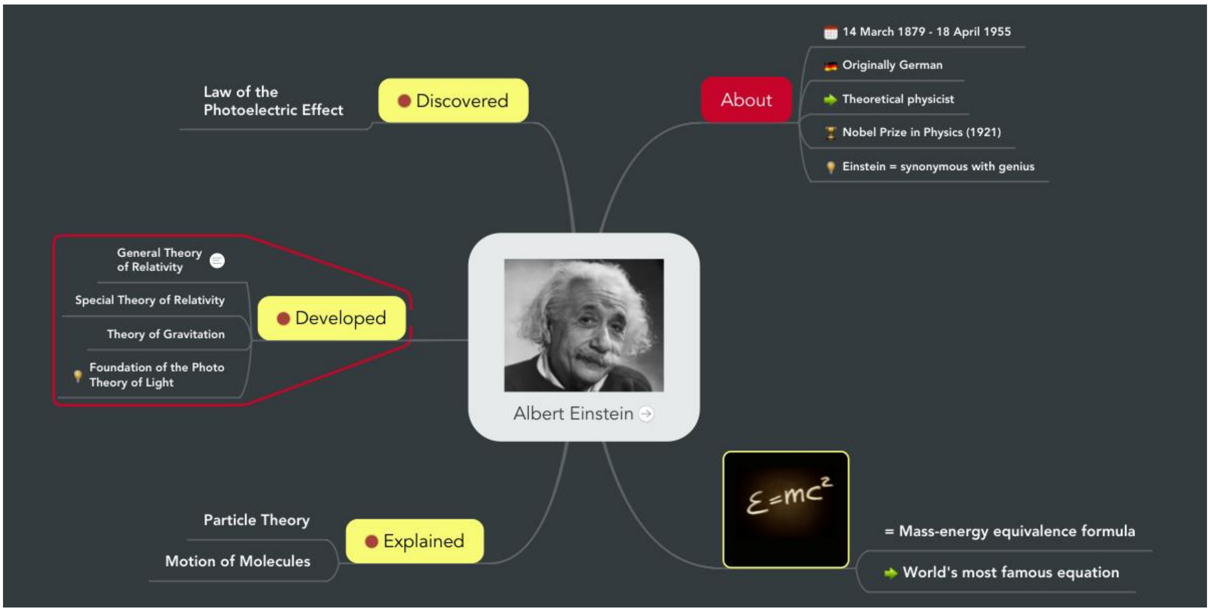
- Hydrostatics is the branch of fluid mechanics that studies (incompressible) fluids at rest, i.e. fluids that have no motion.
- Hydrodynamics, as one might guess, is the study of fluids in motion. Hydrostatics is the branch of fluid mechanics that studies (incompressible) fluids at rest, i.e., fluids have no motion. Hydrodynamics, as one might guess, is the study of fluids in motion.

**Important Questions**

1. State the laws of flotation
2. Define Centre of Pressure
3. Calculate the centre of pressure for a rectangular lamina.
4. Explain stability of floating bodies.
5. Explain the determination of metacentric height of a ship by experiment.

# Unit - V

## Mind Map



**Objectives**

Realize the fundamentals of Special theory of Relativity and the related experiments.

**5. Introduction:**

Classical or Newtonian mechanics deals with the motions of bodies travelling at velocities that are very much less than the velocity of light. According to it, the three fundamental concepts of Physics, viz., space, time and mass are all absolute and invariant.

**Concept of space.** Newton assumed that space is absolute and "exists in itself, without relation to anything external and remains unaffected under all circumstances". This means to say that the length of an object is independent of the conditions under which it is measured, such as the motion of the object or the experimenter.

**Concept of time.** According to Newton, time is absolute 'by its very nature flowing uniformly without reference to anything external'. Hence there is a universal time flowing at a constant rate, unaffected by the motion or position of objects and observers. This implies two things:

(l) The interval of time between two events has the same value for all observers, irrespective of their state of motion.

(2) If two events are simultaneous 'for an observer, they are simultaneous for all observers, irrespective of their state of motion, i.e., *simultaneity is absolute*.

**Concept of mass.** In Newtonian mechanics,

(l) The mass of a body does not depend on the velocity of its motion.

(2) The mass of an isolated system of bodies does not change with any processes occurring within the system (*law of conservation of mass*)

**5.1 FRAME OF REFERENCE:**

A system of co-ordinate axes which defines the position of article in two- or three-dimensional space is called a frame of reference. The simplest frame of reference is the familiar Cartesian system of co-ordinates, in which the position of the particle is specified by its three co-ordinates x,y,z, along the three perpendicular axes. In Fig we have indicated two observers O and O' and a particle P. These observers use frames of reference XYZ and X' Y' Z', respectively. If O and O' are at rest, they will observe the same motion of P. But if O and O' are in relative motion, their observation of the motion of P will be different.

Unaccelerated reference frames in uniform motion of translation relative to one another are called *Galilean frames or inertial frames*.

Accelerated frames are called *non – inertial* frames.

**Definitions**

**1. Inertial frame of reference:**

An *inertial frame of reference* is one in which Newton's first law of motion holds. In such a frame, an object at rest remains at rest and an object in motion continues to move at constant velocity (constant speed and direction) if no force acts on it. Any 'frame of reference that moves at constant velocity relative to an inertial frame is itself an inertial frame.

**Special theory of relativity** deals with the problems that involve inertial frames of reference.

**Non - Inertial frame of reference:**

A *non-inertial frame of reference* is the one in which the Newton's laws of motion are not valid i.e., a body is accelerated when no external force acts on it.

### 5.3 Newtonian Principle of Relativity

**Statement:** *Absolute motion, which is the translation of a body from one absolute place to another absolute place, can never be detected. Translatory motion can be perceived only in of motion relative to other material bodies.*

**Explanation:** This implies that if we are drifting along at a uniform speed in a closed spaceship all the phenomena observed and all the experiments performed inside the ship will appear to be the same as if the ship were not in motion. This means that the fundamental physical laws and principles are identical in all inertial frames of reference. This is the concept of Newtonian relativity.

### 5.4 Galilean Transformation Equations

Let S and S’ be two inertial frames. Let S be at rest and S' move with uniform velocity v along the positive X direction. We assume that v < < c. Let the origins of the two frames coincide at t = 0. Suppose some event occurs at the point P. The observer O in frame S determines the position of the event O the coordinates x, y, z. The observer O' in frame S' determines the position of the event by the coordinates x', y', z'. There is no relative motion between S and S' along the axes of Y and Z. Hence, we have y = y' and z = z'. Let the time proceed at the same rate in both frames.

The distance moved by S' in the positive X-direction in time t = vt. So, the X coordinates of the two frames differ by Vt. Hence, x' = x — vt.

Then the transformation equations from S to S' are given by,

$x' = x - vt$ ---------(1) $y' = y$ -----------(2) $z' = z$ ----------(3) $t' = t$ -----------(4)

Notes:

(l) The *inverse transformation equations* (from S' to S) are

x = x' + vt   y’ = y   z = z' and t = t'.

(2) In general, the transformation of velocities from one to the other system is obtained by taking time derivatives. When the relative motion of the two frames is confined to the X-direction, the transformation becomes,

$$\frac{dx'}{dt} = \frac{dx}{dt} - v \text{ i.e., } u' = u - v; \ \frac{dy'}{dt} = \frac{dy}{dt}; \frac{dz'}{dt} = \frac{dz}{dt}.$$

(3) Let $a$ and $a'$ be the accelerations of the particle in $S$ and $S'$. We have $a = \frac{du}{dt}$ and $a' = \frac{du'}{dt}$.

We have seen above that $u' = u - v$.

$\therefore \ \frac{du'}{dt} = \frac{du}{dt}$   (since $v$ is constant)

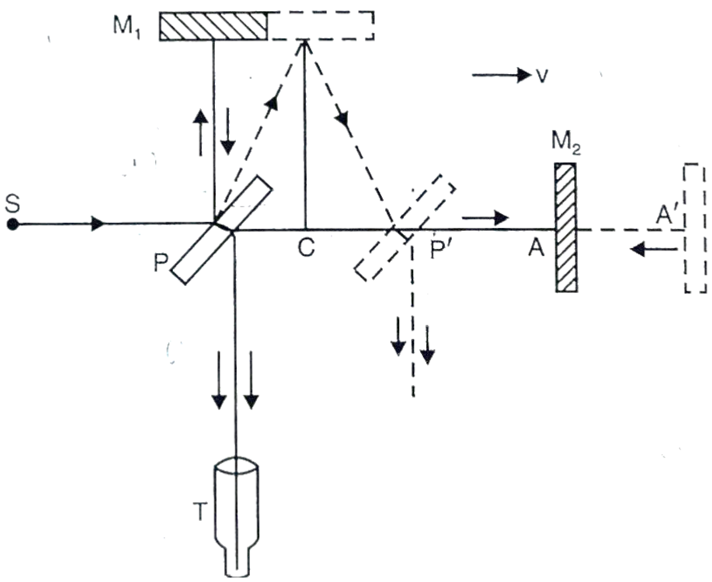
Or a’= a i.e., the accelerations, as measured by the two observers in the two frames, are the same. Hence we say that acceleration is invariant under *Galilean transformation*

### 5.5 The Ether Hypothesis

A material medium is a necessity for the propagation of waves. It was considered that light propagates through ether as the sound waves propagate through air. Ether pervades all space. An interesting question is whether relative motion between the earth and ether can be detected. If such a motion can be detected, we can choose a fixed frame of reference in a stationary ether. Then we can express all motion relative to this frame. In 1887, Michelson and Morley, set out to measure the relative velocity of earth with respect to the ether. The principle of the experiment lies in noting the shift in fringes in the Michelson interferometer due to the difference in time taken by light to travel along and opposite the direction of motion of the earth. The time taken by a beam of light to travel along the direction of motion of earth is greater than that to travel distance opposite to the direction of motion of the earth. Surprisingly, despite best efforts, the presence of ether could not be detected.

**5.6 The Michelson – Morley Experiment:**

A beam of light from a monochromatic source S fall on a half-silvered glass plate P, placed at an angle of 45 $^0$ to the beam. The incident beam is split up into two parts by P (Fig). The reflected portion travels in a direction at right angles to the incident beam, falls normally at B on the plane mirror $M_1$ and is reflected back to P. It gets refracted through P and enters the telescope T. The transmitted portion travels along the direction of the initial beam, falls normally on mirror M. at A and is reflected back to P. After reflection from the back surface of P, it enters the telescope T. The two reflected beams interfere and the interference fringes are viewed with the help of the telescope T. The beam reflected upwards to $M_1$ traverses the thickness of plate P thrice whereas the beam refracted on to mirror $M_2$ traverses P only The effective distance of the mirrors $M_1$ and $M_2$ from the plate P is made to be the same by the use of a compensating plate not shown in figure.




The whole apparatus was floating on mercury. One arm (PA) was pointed in the direction of earth's motion round the sun and the other (PB) was pointed perpendicular to this motion. The paths of the two beams and the positions of their reflections from $M_1$ and $M_2$ will be as shown by the dotted lines. Assume that the velocity of the apparatus (or earth) relative to fixed ether is r on the direction PA. The relative velocity of a light ray travelling along PA is (c-v) while its value would be (r+v) for the returning ray. Let PA = PB = d

Time taken by light to travel from P to A = d/(c-v)

Time taken by light to travel from A to A = d/(c+v)

Total time taken by light to travel from P to A and back

$$t = \frac{d}{c-v} + \frac{d}{c-v} = \frac{2cd}{c^2 - v^2} = \frac{2d}{c}\left(1 + \frac{v^2}{c^2}\right) \text{ ------------------(1)}$$

Now, consider the ray moving upwards from P to B. It will strike the mirror $M_1$ not at B but at B' due to the motion of the earth. If $t_1$ is the time taken by the ray starting from P to reach $M_1$. Then PB' = $ct_1$ and BB' = $vt_1$.

The total path of the ray until it returns to P = PB'P'

Now, consider the ray moving upwards from P to B. It will strike the mirror $M_1$ not at B but at B' due to the motion of the earth. If $t_1$ is the time taken by the ray starting from P to reach $M_1$, then PB' = $ct_1$ and BB' = $vt_1$.

The total path of the ray until it returns to P = PB'P'

Now PB'P' = PB' + B'P' = 2PB', Since PB' = B'P'.

$$(PB)^2 = PC^2 + (CB)^2 = (BB')^2 + (PB)^2$$

i.e., $c^2t_1^2 = v^2t_1^2 + a^2$.

Total time taken by the ray to travel the whole path PB'P'

$$t' = 2t_1 = \frac{2d}{\sqrt{c^2 - v^2}} = \frac{2d}{c\sqrt{1-(v^2/c^2)}} = \frac{2d}{c}\left(1 + \frac{v^2}{2c^2}\right) \text{ ----------- (2)}$$

Clearly t' < t. The time differences

$$\Delta t = t - t' = \frac{2d}{c}\left(1 + \frac{v^2}{c^2}\right) - \frac{2d}{c}\left(1 + \frac{v^2}{2c^2}\right) = \frac{2d}{c} \times \frac{v^2}{2c^2} = \frac{dv^2}{c^2}.$$

The distance travelled by light in time $\Delta t = c \times \Delta t = \frac{dv^2}{c^2}$.

This is the path difference between the two parts of the incident beam. If the apparatus is turned through 90˚, the path difference between the two beam becomes $2dv^2/c^2$. Michelson an Morley expected a fringe shift of about 0.4 in their apparatus when it was rotated through 90 °and they believed that they could detect a shift as small as 0.01 of a fringe. But, in the experiment no displacement of the fringes was observed. They repeated the experiment at different points on the earth's surface and at different seasons of the year without detecting any measurable shift in fringes. *This negative result suggests that it is impossible to measure the speed of the earth relative to the ether.* Therefore, the effects of ether are undetectable. Thus, all attempts to make ether as a fixed frame of reference failed.

**Explanation of the negative result:**

The negative result of the Michelson – Morley experiment can be explained by the following three explanations.

- The earth dragged along with it the ether in its immediate neighbourhood. Thus, there was no relative motion between the earth and ether. This is the explanation proposed by Michelson himself.
- Lorentz and Fitzgerald put fourth the suggestion that there was concentration of bodies along the direction of their motion through the ether. Let $L_0$ be the length of the body when at rest. If it is moving with a speed v parallel to its length, the new length L is $L_0\sqrt{1-(v^2/c^2)}$. In the experiment discussed above, the distance PB will remain unchanged. Distance PA will get shortened to $d\sqrt{1-(v^2/c^2)}$. If d were replaced by $d\sqrt{1-(v^2/c^2)}$ in equation (1), t and t' will be the same and there will be no time difference expected. This contraction hypothesis easily explains why the Michelson – Morley experiment gave a negative result.
- The proper explanation for the negative result of the Michelson – Morley experiment was given by Einstein. He concluded that the velocity of light in space is a universal constant. This statement is called the *principle of constancy of the speed of light.* The speed of light is c rather than (c+v) in any frame.

**5.7 Special Theory of Relativity**

Einstein propounded the special theory of relativity in 1905. The special theory deals with the problems in which one frame of reference moves with a constant linear velocity relative to another frame of reference.

**Postulates of special theory of relativity**

- ✓ *The laws of Physics are the same in all inertial frames of reference.*
- ✓ *The velocity of light in free space is constant. It is independent of the relative motion of the source and the observer.*

Explanation:

1. The first postulate expresses the fact that, since it is impossible to perform an experiment which measures motion relative to a stationary ether, no unique stationary frame of reference can be discovered. Since there is no favoured 'rest' frame of reference, all systems moving with constant velocity must be on equal footing. We cannot discuss absolute motion. We can discuss only relative motion.
2. We know that the velocity of light is not constant under Galilean transformations. But according to the second postulate, the velocity of light is the same in all inertial frames. Thus, the second postulate is very important and only this postulate is responsible to differentiate the classical theory and Einstein's theory of relativity.

**5.8 Lorentz Transformation**

We have to introduce new transformation equations which are consistent with the new concept of the invariance of light velocity in free space. The new transformation equations were discovered by Lorentz, and are known as "***Lorentz transformations***".

**Derivation.** Consider two observers O and O' in two systems S and S'. System S' is moving with a constant velocity v relative to system S along the positive X-axis. Suppose we make measurements of time from the instant when the origins of S and S' just coincide, *i.e.*, $I = 0$ when O and O' coincide. Suppose a light pulse is emitted when O and O' coincide. The light pulse produced at t = 0 will spread out as a growing sphere. The radius of the wave front produced in this way will grow with speed c. After a time t, the observer O will note that the light has reached a point P (x,y,z) as shown in fig. The distance of the point P is given by r = ct. From figure $r^2 = x^2 + y^2 + z^2$.

Hence, $x^2 + y^2 + z^2 = c^2t^2$ ------------------(1)

Similarly, the observer $O'$ will note that the light has reached the same point $P$ in a time $t'$ with the same velocity c. So $t' = ct'$.

$\therefore\ x'^2 + y'^2 + z'^2 = c^2t'^2$ ------------------(2)

Now, equations (1) and (2) must be equal since both the observers are at the centre of the same expanding wavefront.

$\therefore\ x^2 + y^2 + z^2 - c^2t^2 = x'^2 + y'^2 + z'^2 - c^2t^2$ ------------------(3)

Since there is no motion in the $Y$ and $Z$ directions, $y' = y$ and $z' = z$.

$\therefore$ Equation (3) becomes,

$x^2 - c^2t^2 = x'^2 - c^2t'^2$ ------------------(4)

The transformation equation relating to x and x' can be written as,

$x' = k(x - vt)$ ------------------(5)

Here, $k$ is a constant.

The reason for trying the above relation is that, the transformation must reduce to Galilean transformation for low speed ($v << c$).

Similarly, let us assume that

$t' = a(t - bx)$ ------------------(6)

Here, $a$ and $b$ are constants.

Substituting these values for $x'$ and $t'$ in equation (4), we have,

$$x^2 - c^2t^2 = k^2(x - vt)^2 - c^2a^2(t - bx)^2$$

i.e., $x^2 - c^2t^2 = (k^2 - c^2a^2b^2)x^2 - 2(k^2v - c^2a^2b)xt - \left(a^2 - \frac{k^2v^2}{c^2}\right)c^2t^2$ ----(7)

Equating the coefficients of corresponding terms in equation (7),

$$k^2 - c^2a^2b^2 = 1$$
$$k^2v - c^2a^2b = 0$$
$$a^2 - \frac{k^2v^2}{c^2} = 1$$
------------------(8), (9) and (10)

Solving the above equations for $k, a$ and $b$, we get,

$$k = a = \frac{1}{\sqrt{1-(v^2/c^2)}}$$ ------------------(11)

And $b = v/c^2$ ------------------(12)

Substituting these values of $k, a$ and $b$ in (5) and (6) we have,

$$x' = \frac{x - vt}{\sqrt{1 - (v^2/c^2)}} \text{ and } t' = \frac{t - (vx/c^2)}{\sqrt{1 - (v^2/c^2)}}$$

Therefore, the Lorentz transformation equations are

$$x' = \frac{x - vt}{\sqrt{1 - (v^2/c^2)}}; y' = y; z' = z \text{ and } t' = \frac{t - (vx/c^2)}{\sqrt{1 - (v^2/c^2)}} \ldots (13)$$

The inverse Lorentz transformation equations are obtained by interchanging the coordiantes and replacing v by – v in the above

$$\mathrm{x} = \frac{x' + vt'}{\sqrt{1 - (v^2/c^2)}}; \mathrm{y} = y'; \mathrm{z} = z' \text{ and } \mathrm{t} = \frac{t' + (vx'/c^2)}{\sqrt{1 - (v^2/c^2)}} \ldots (14)$$

These equations convert measurements made in frame S’ into those in frame S.

**5.9 Length Contraction**

Consider two coordinate systems S and S’ with their X-axes coinciding at time t O. S' is moving with a uniform relative speed v with respect to S in the positive *direction, Imagine a rod (AB), at rest relative to S'

Let $x'$, and $x_2'$ be the coordinates or the ends of the rod at any instant of time in $S'$. Then,

$$I_0 = x_2{}' - x_1{}'$$

since the rod is at rest in frame $S'$.

Similarly, let $x_1$ and $x_2$ be the coordinates of the ends of the rod at the same instant of time in $S$. Then

$$l = x_2 - x_1$$

$l$ is the length of the rod, measured relative to $S$.

According to Lorentz transformations,

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - (v^2/c^2)}}$$

$$\text{and } x_1' = \frac{x_1 - vt}{\sqrt{1 - (v^2/c^2)}}$$

Subtracting equation (4) from (3)

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-(v^2/c^2)}} \text{ or } l_0 = \frac{l}{\sqrt{1-(v^2/c^2)}}$$

$$\therefore\ l = l_0\sqrt{1-(v^2/c^2)}$$

From equation (5) we see that $l<l_0$. Therefore, to the observer in S it would appear that the length of the rod (in S') has contracted by the factor $\sqrt{1-(v^2/c^2)}$.

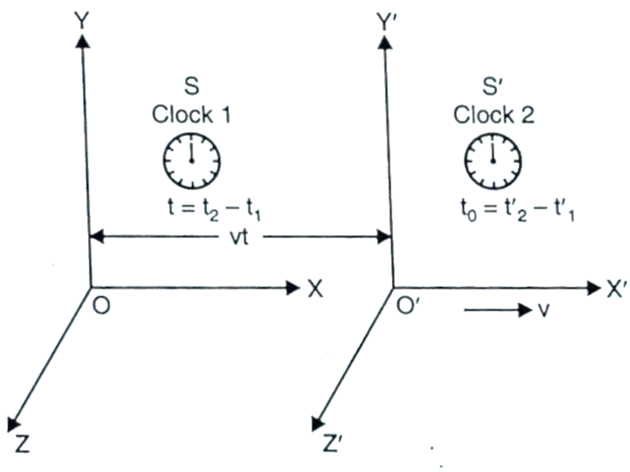
For example, a body which appears to be spherical to an observer at rest relative to it, will appear to be an oblate spheroid to a moving observer. Similarly, a square and a circle in one appear to the observer in the other to be a rectangle and an ellipse respectively.

**Notes:**

- ✓ The proper length of an object is the length determined by an observer at rest with respect to the object. In the above case, 10 is the proper length.
- ✓ The shortening or contraction in the length of an object along its direction of motion is known as the Lorentz-FitzgeraId contraction. There is no contraction in a direction perpendicular to the direction Of motion.
- ✓ The contraction becomes appreciable only when v C.
- ✓ The contraction is reciprocal, i.e., if two identical rods are at rest—one in S and the other in S', each of the observers finds that the other is shorter than the rod of his own system.

**Time Dilation:**

Imagine a gun placed at the position (x',y',z') in S'.



Suppose it fires two shots at times $t_1$' and $t_2$' measured with respect to S'. In S' the clock is at rest relative to the observer. The time interval measured by a clock at rest relative to the observer is called the proper time interval. Hence, to = t2' — ti' is the time interval between the two shots for the observer in S'.

Since the gun is fixed in S', it has a velocity v with respect to S in the direction of the positive X-axis. Let t = t2 — tl represent the time interval between the two shots as measured by an observer in S.

From inverse Lorentz transformations, we have

$$t_1 = \frac{t_1' + vx'/c^2}{\sqrt{1-(v^2/c^2)}} \text{ and } t_2 = \frac{t_2' + vx'/c^2}{\sqrt{1-(v^2/c^2)}}$$

$$\therefore\ t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1-(v^2/c^2)}} \text{ or } t = \frac{t_0}{\sqrt{1-(v^2/c^2)}}$$

$$\text{or } t > t_0.$$

Thus, the time interval, between two events occurring at a given point in the moving frame $S'$ appears to be longer to the observer in the stationary frame $S$; i.e., a stationary clock measures a longer time interval between events occurring in a moving frame of reference than does a clock in the moving frame. This effect is called time dilation.

**The Twin Paradox:**

Consider two exactly identical twin brothers. Let one of the twins go to a long space journey at a high speed in a rocket and the other stay behind on the earth. The clock in the moving rocket will appear to go slower than the clock on the surface of the earth (in accordance with $t = \frac{t_0}{\sqrt{1-(v^2/c^2)}}$). Therefore, when he returns to the earth, he will find himself younger than the twin who stayed behind on the earth!

**Summary**

- **Relativity**, Concept in physics that measurements change when considered by observers in various states of motion. In classical physics, it was assumed that all observers anywhere in the universe would obtain identical measurements of space and time intervals.
- According to relativity theory, this is not so; all measurements depend on the relative motions of the observer and the observed. There are two distinct theories of relativity, both proposed by Albert Einstein.
- The special theory of relativity (1905) developed from Einstein's acceptance that the speed of light is the same in all reference frames, irrespective of their relative motion.

- It deals with non-accelerating reference frames, and is concerned primarily with electric and magnetic phenomena and their propagation in space and time.

**Important Questions**

1. What are inertial Frames?
2. Write the postulates of special theory of relativity.
3. Differentiate Galilean and Lorentz transformations.
4. Derive the expression for Einstein's most energy equivalence.
5. Explain the Michelson and Morley experiment. Give its importance.

**Sample Question Paper**

| F–1568 |
|---|

| Sub. Code |
|---|
| 7BPH1C2 |

**B.Sc. DEGREE EXAMINATION, APRIL 2019.**

**First Semester**

**Physics**

**MECHANICS AND RELATIVITY**

**(CBCS – 2017 onwards)**

Time : 3 Hours　　　　Maximum : 75 Marks

**Section A**　　　　$(10 \times 2 = 20)$

Answer **all** questions.

1. State limiting friction.

   வரம்பு உராய்வு என்றால் என்ன?

2. Define coefficient of friction.

   உராய்வுக் குணகத்தை வரையறு.

3. Define torque.

   வரையறு திருப்பு விசை.

4. State the Kepler's second low of planetary motion.

   கோள்களின் இயக்கத்திற்கான கெப்ளரின் இரண்டாம் விதியைக் கூறு.

5. Define torque.

   திருப்பு விசையை வரையறு.

6. What is radius of gyration?

   சுழற்சி ஆரம் என்றால் என்ன?

7. What is meta centre?

மிதவைக் காப்பு மையம் என்றால் என்ன?

8. State Bernoulli's theorem.

பெர்னோலியின் தேற்றத்தினைக் கூறு.

9. What is time dilation?

கால நீட்டிப்பு என்றால் என்ன?

10. Write the expression for Einstein's mass energy relation.

ஐன்ஸ்டீனின் நிறை ஆற்றல் தொடர்பிற்கான சமன்பாட்டை எழுதுக.

**Section B** (5 × 5 = 25)

Answer **all** the questions, choosing either (a) or (b).

11. (a) Explain working of clutch.

விடு பற்றியின் செயல்பாட்டை விவரி.

Or

(b) Calculate the centre of gravity of a hollow hemisphere.

ஒரு உள்ளீடற்ற அரைக் கோளத்தின் ஈர்ப்பு மையத்தைக் கணக்கிடுக.

12. (a) Derive an expression for gravitational potential and explain.

ஈர்ப்பழுத்தத்திற்கான கோவையை வருவித்து, விளக்குக.

Or

(b) Calculate orbital velocity of geostationary satellite.

செயற்கைக்கோளின் சுற்றுத் திசைவேகத்தைக் கணக்கிடுக.

13. (a) Derive an expression for kinetic energy of rotating body.

சுழலும் பொருளின் இயக்க ஆற்றலுக்கான கோவையை வருவி.

Or

(b) State and explain parallel axis theorem.

இணையச்சுத் தேற்றத் துணைக் கூறி விளக்குக.

14. (a) Explain experimental determination of metacentric height of a ship.

ஒரு கம்பியின் மிதவைக்காப்புயரத்தினைக் கணக்கிடும் சோதனையை விவரி.

Or

(b) Explain stability of floating bodies.

மிதக்கும் பொருளின் நிலைத் தன்மையை விவரி.

15. (a) Deduce Einstein's mass-energy relation.

ஐன்ஸ்டீனின் நிறை ஆற்றல் தொடர்பினை வருவி.

Or

(b) Give the postulates of special theory of velocity.

சிறப்புச் சார்பியல் கொள்கைக்கான எடுகோள்களைத் தருக.

## Section C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Deduce an expression for centre of gravity of solid cyclindes.

திண்ம உருளையின் ஈர்ப்பு மையத்திற்கான கோவையை வருவி.

17. Explain the Boy's method for finding 'G'.

'G'ன் மதிப்பைக் கணக்கிடுவதற்கான பாய்ஸ் சோதனையை விவரி.

18. Explain theory of compound pendulum and also determination of acceleration due to gravity.

கூட்டு ஊசலின் கொள்கையை விவரித்து அதன் மூலம் புவியீர்ப்பு முடுக்கத்தினை காணும் விதத்தினையும் விளக்கு.

19. Calculate the center of pressure of rectangular Laminae inside a uniform density liquid.

ஒரு படித்தான திரவத்தில் மூழ்கியவாறு அமைந்த செவ்வக வடிவத் தகட்டின் அழுத்த மையத்தைக் கணக்கிடு.

20. Explain Michelson Morley experiment and its importance.

மைக்கெல்சன் மார்லே சோதனை மற்றும் அதன் முக்கியத்துவத்தை விளக்குக.

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